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ABSTRACT

This booklet is a collection of puzzles, games, and investigations. All that children need are some stones or shells, on some of which they must write numerals. For playing with the whole class, the game or puzzles may be marked out on the floor or in sand; in that case, larger objects such as small rocks and empty tins may be used. Children are encouraged to use an experimental, trial-and-error approach at first, but the goal is to develop solution strategies. (MNS)

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A SOURCE BOOK OF SIMPLE PUZZLES
AND PROBLEMS

WILLIAM GIBBS

MAY 1982

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INTRODUCTION

All you need for these puzzles, games and investigations are some stones or shells. Collect some small smooth pebbles from the beach, a handful of white ones and a handful of grey or black ones. For the puzzles which need numbered stones use the smooth white ones and write numbers on them with a pencil or felt tip.

The best shells to use are little ones, for example different kinds of cowries. For sliding puzzles the 'cats eyes' (or 'lids') which have a smooth flat side are good. It is easy to write numbers on these too.

If you want to play the game or puzzle with the whole class then it is a good idea to mark out the puzzle or game diagram on the floor, or in the sand, or on a loose blackboard laid on the floor. Then you need large objects instead of small stones. Big smooth river stones or empty tins are easy to handle.

Children will enjoy playing the games in this book. Perhaps after they have learnt them in school they will play them with their friends after school or with their family at home. But to enjoy these puzzles children will need to be able to solve them! If the puzzle is too difficult the child will easily get frustrated. In this book I have tried to start with easy puzzles so that every child can begin to feel 'I can do this! Then they will be prepared to work longer on harder puzzles and will gain in determination and perseverance.

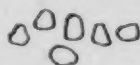
When children start these puzzles they may use trial and error to find a solution. As they do more encourage them to think; 'what will happen if I move here?' Children will need to move the stones at first. Later they will be able to make pictures in their mind of what will happen. This is an important step forward in learning.

In many sections in this book after several puzzles there is an 'investigation'. This is to challenge the more able and older children to think of the rules and patterns that lie behind the puzzles and games. First encourage the children to invent ways of recording their answers. Often they will have to invent their own symbols and signs. Then it is easier to see the pattern or general rule.

If the children particularly enjoy one kind of puzzle then challenge them to make up similar ones of their own. There is a great chance here to encourage creativity in the child. I think, too, you will find that children enjoy solving puzzles set by their friends more than those set by you!

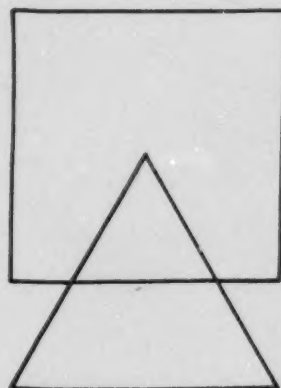
1 SUM PUZZLES

You will need 6 stones



Puzzle 1

Can you place the 6 stones on the diagram so that 5 are in the square and 4 in the triangle?



Puzzle 2

6 stones
Put 3 in the square
and 4 in the triangle.

Puzzle 3.

6 stones
Put 5 in the square
and 5 in the triangle.

Puzzle 4

6 stones
Put 3 in the square and
5 in the triangle

Puzzle 5

6 stones
Put 1 in the square and
6 in the triangle.

Puzzle 6

6 stones

Put 5 in the square and
2 in the triangle

Puzzle 7

Put 6 stones in the square and
6 in the triangle.

More Sum Puzzles

Puzzle 1 10 stones.

Put 7 in the circle and
7 in the triangle.

Puzzle 2 10 stones.

Put 8 in the circle and
5 in the triangle.

Puzzle 3 10 stones.

Put 6 in the circle and
8 in the triangle.



Make up some puzzles like these of your own.

Investigation

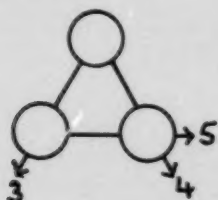
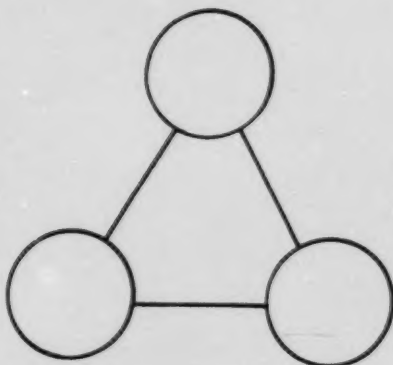
- (1) If you had 100 stones, could you arrange them so that there are 67 in the circle and 59 in the triangle?
- (2) Can you find a general way of solving these puzzles no matter how big the numbers are?

2

TRIANGLE PUZZLES

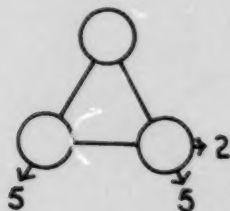
You need 6 stones.

Puzzle 1. Place the 6 stones in the circles so that the total of stones on each side of the triangle is 3, 4 and 5.

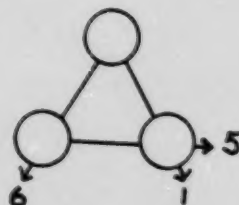


Still using 6 stones make the total shown.

Puzzle 2

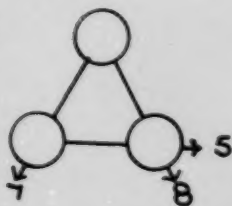


Puzzle 3

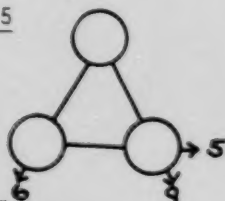


Now take 10 stones. Make the totals shown.

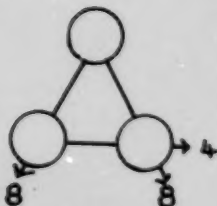
Puzzle 4



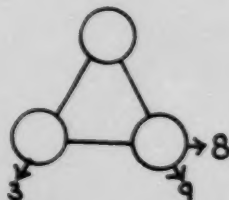
Puzzle 5



Puzzle 6

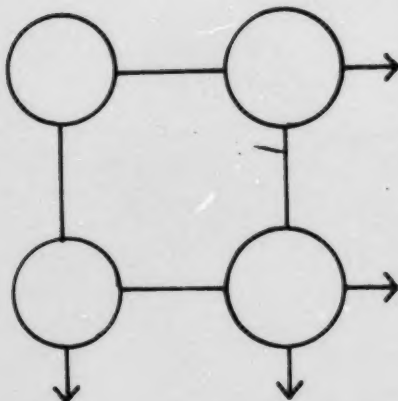


Puzzle 7



Investigation

- (1) How many different puzzles like this could we make using 8 stones? What is the connection between the totals and the number of stones?
- (2) There is a simple way of solving these puzzles similar to that used in Sum puzzles. Can you find it?
- (3) Invent a similar puzzles using a square diagram. Is there just one solution to each puzzle?



3

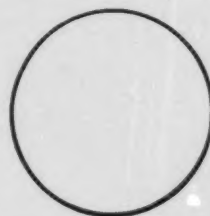
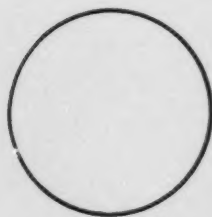
PILES

1. Simple Game:

Take 12 stones. Make 3 piles like this:



Now can you move 1 stone so that each pile has 4 stones.



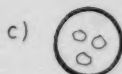
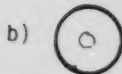
2. Harder Games:

Take 12 stones. Make 3 piles like this:



This time you have 2 moves. On the first move you can move 1 stone. On the second move you can move 2 stones from one pile to another. In 2 moves can you rearrange the stones into 3 equal piles.

Now using the same rules, try these:



3. Harder Still:

This time you are allowed 3 moves

First move, move 1 stone.
Second move, move 2 stones.
Third move, move 3 stones.

Start with 12 stones arranged in piles of 2, 4 and 6.



Try and make 3 piles of 4 stones after the 3 moves.



Using the same rules, start with piles of:

a) 1, 3, 8

b) 2, 8, 2

c) 3, 4, 5

d) 3, 6, 3

e) 1, 10, 1

f) 1, 4, 7

g) 0, 3, 9

h) 0, 5, 7

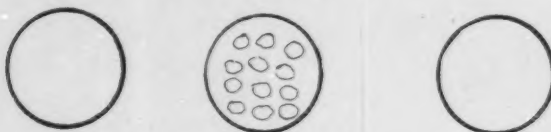
3 Piles Puzzles
Puzzle 1

Start with 3 piles with 4 stones in each



Move 1 stone on the first move. Move 2 stones on the second move. And so on. Can you continue for 12 moves?

Puzzle 2 Starting with 3 piles of 4 and the same moves, what is the fewest number of moves it takes to get to this position?



Puzzle 3 Starting with three piles of 12, 0, 0 can you get back to 4, 4, 4 using the same kind of moves.

Game of Piles. 2 players

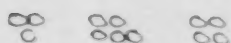
Make 3 piles with 4 stones in each

On the first move the first player must move 1 stone.
 On the next move the second player must move 2 stones.
 On the third move the first player must move 3 stones.
 On the fourth move the second player must move 4 stones.
 and so on.

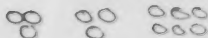
On each move you may only move stones from one pile to another.

The first player who cannot move loses.

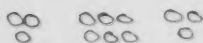
Example



First player moves 1



2nd player moves 2



3rd player moves 3

and so on.

INVESTIGATION

- (1) How many different starting arrangements with 12 stones can be equalized to 3 piles of 4 stones in just 3 moves?
- (2) Are there any starting arrangements which cannot be equalized in 3 moves?
- (3) A further problem. Can we choose the number of stones so that all starting arrangements can be equalized in 3 moves? in 4 moves?
- (4) Can you find a general rule for the most stones that can be equalized from any starting arrangement in N moves?

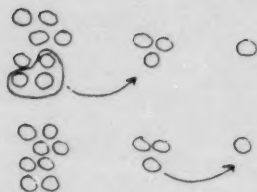
4

HEAPS

12 stones



Start with 3 heaps of stones; 8 in one, 3 in another and 1 in another.



On each move you are allowed to add to a heap the number of stones already in that heap. For example you could take 3 from the heap of eight to add to the heap of 3. Or you could take 1 from the heap of three to add to the heap of 1.

In just 3 moves can you make 3 heaps with 4 stones in each.

Now try starting with these a) 2, 9, 1

b) 2, 7, 3

24 stones



Using the same rules as above, can you equalize the heaps in just 3 moves?

Try starting with a) 3, 8, 13

b) 7, 8, 9

c) 2, 7, 15

Hard Ones

Using the same rules, but using more moves, try and equalize these heaps.

a) 16, 5, 3

b) 13, 9, 2

c) 10, 9, 5

d) 14, 5, 5

INVESTIGATION

- a) Starting with 12 stones, how many different starting arrangements can be equalised in 3 moves? (Hint : reverse the problem)
- b) Ignoring starting arrangements which contain 0 in one pile and are clearly impossible, are there any other arrangements of 12 stones, in three piles, which can not be equalised in 4 moves? In any number of moves?
- c) How about 24 stones? Are there arrangements that cannot be equalised if you are allowed any number of moves?
- d) What number of stones should be chosen so that all arrangements can be equalised?

5 ODDS & EVENS

Can you put 5 stones in two squares so that there is an odd number of stones in each square?

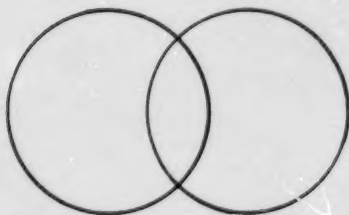
Can you put 9 stones in four squares so that there is an odd number of stones in each square?

These two puzzles are quite tricky. We will come back to them later and if you can not solve them now maybe you will be able to then.

The puzzles and games in this section are all to do with odd and even numbers. Remember that Even numbers can be divided by 2 exactly. Examples of even numbers are 2, 4, 6, 8, 10, 12, 56, 198, 400.

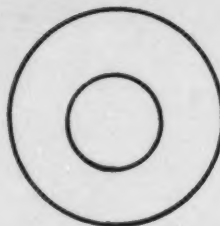
Odd numbers are all the other counting numbers, numbers like 1, 3, 5, 7, 35, 163. Now try the puzzles and games.

Puzzle 1. Take 5 stones. Put them in the two circles so that there is an odd number in each circle.



Puzzle 2

Take 5 stones and put them in the two circles so that there is an odd number in both.



Now can you do the puzzles at the start of this chapter?

Puzzle 3

Take 5 stones.
Put an odd number
in A, B and C.

Puzzle 4

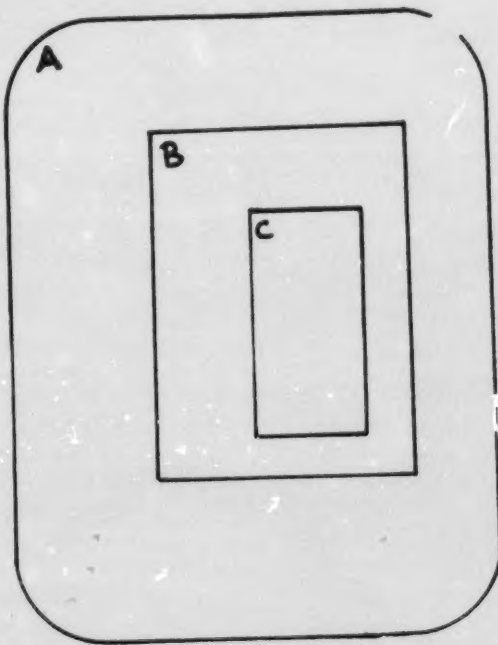
Still use 5 stones.
Put an odd number in A.
Put an even number in B.
Put an odd number in C.

Puzzle 5

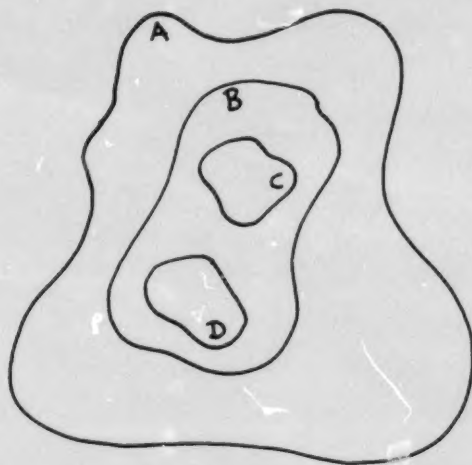
Put an odd number in A.
Put an even number in B.
Put an even number in C.

Puzzle 6

Put an odd number in A.
Put an odd number in B.
Put an even number in C.



Now try this with 7 stones:



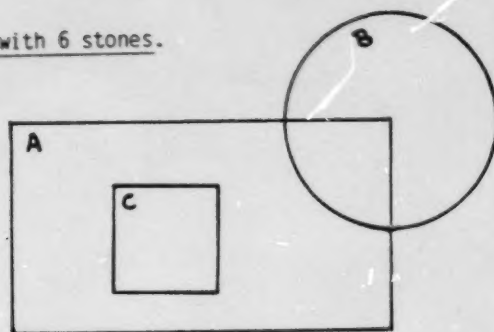
(A) Put stones so that the
number in A is odd,
in B is odd,
in C is odd,
in D is odd.

(B) Still using 7 stones put
them so that the
number in A is odd,
in B is odd,
in C is even,
in D is even.

(C) So that the number
in A is odd,
in B is even,
in C is odd,
in D is odd,

(D) So that the number
in A is odd,
in B is odd,
in C is odd,
in D is odd,

Now try this with 6 stones.



- | | |
|--|---|
| (A) So that A is even,
B is even,
C is even. | (B) So that A is odd,
B is odd,
C is odd. |
| (C) So that A is odd,
B is even,
C is odd. | (D) So that A is even,
B is even,
C is odd. |
| (E) So that A is even,
B is odd,
C is odd. | (F) So that A is even,
B is odd,
C is even. |
| (G) So that A is odd,
B is odd,
C is even. | (H) So that A is odd,
B is even,
C is even. |

Now try and solve all these puzzles using 5 stones instead of 6! Can all of them be solved?

Can you make up puzzles like these?

Crossing to the island



You need numbered stones. For the first puzzle use stones numbered

① ② ③ ④ ⑤ ⑥

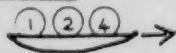
Put all the stones in the village. You have to take all the stones from the village to the island in the canoe. But there are 3 rules.

- (1) The canoe cannot carry more than 3 stones at a time.
- (2) The sum of the numbers in the canoe must be ODD.
- (3) One stone is needed to paddle the canoe back from the island. It must be odd.

Try and get all the stones to the island in 5 journeys.

Example

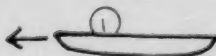
First journey



To the Island ($1 + 2 + 4 = 7$ which is odd)

Leave ② ④ on the island.

Second journey



① brings the canoe back to the village.

Crossing to the Island 2

Now try with 7 stones starting in the village.
They should be numbered 1 to 7.
This time we change the rules a little.

- (1) the same: only 3 stones in the canoe,
- (2) the sum of the numbers in the canoe must be EVEN,
- (3) the canoe must be paddled back by an even number.

Can you get all the stones to the island in 5 journeys?

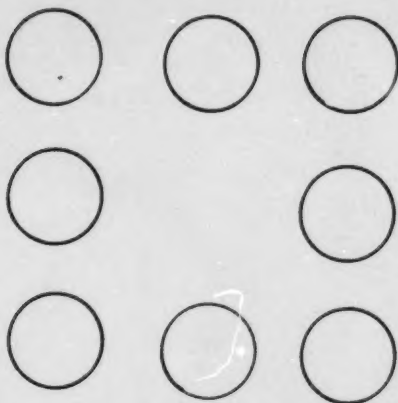
Crossing to the Island 3

This time you start with 8 stones numbered from 1 to 8.
This time you must choose yourself which rule to use. Will you carry an EVEN total or an ODD total? You must not change the rule in the middle. Find out which rule makes the puzzle work.

INVESTIGATION

- (a) If we have 9 stones should we use the EVEN rule or the ODD rule?
- (b) Starting with any number of stones can you work out which rule will get all the stones to the island?

Square Puzzle



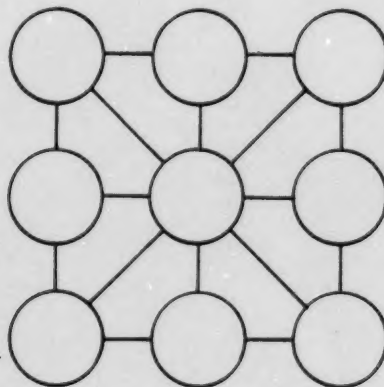
Take 8 stones numbered from 1 to 8.

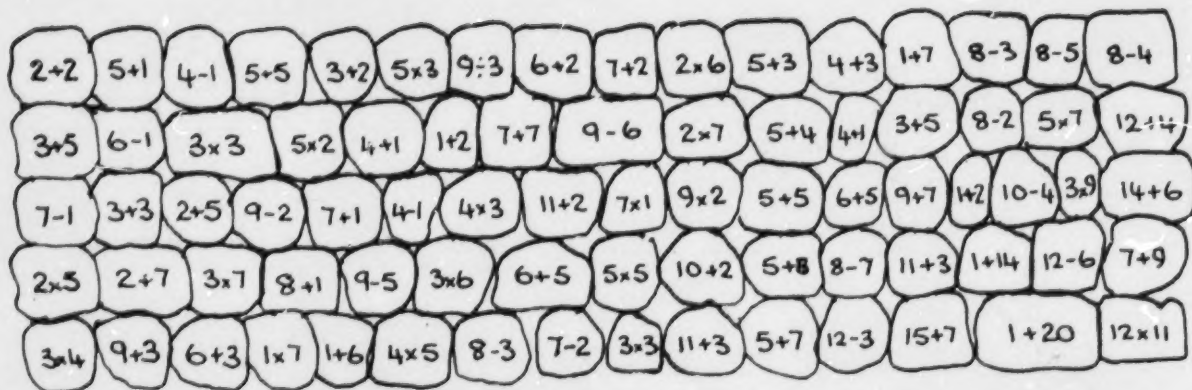


- (1) Place each stone in a circle so that each side of the square adds to an EVEN total.
- (2) Place each stone in a circle so that each side of the square adds to an ODD total.

-
- (3) Now take 9 stones, numbered 1 to 9. Place them so that the sum along each line is ODD.

- (4) Can you place them so that the sum along each line is even?



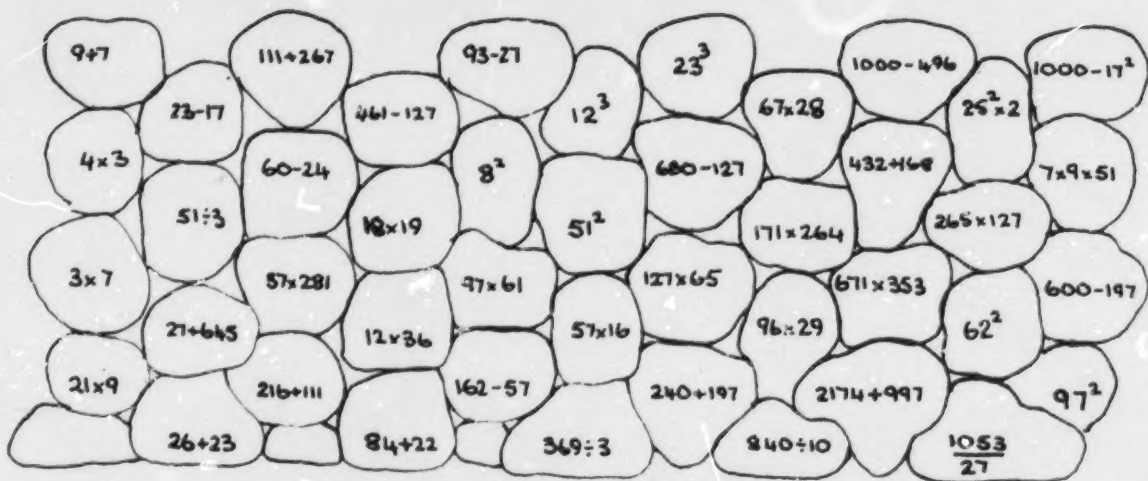


The Even Stone Wall

Put stones on the wall wherever the answer is an even number.

The Odd Stone Wall

Put stones on the wall wherever the answer is an ODD number. You don't have to work out the answers, just think whether they will be odd or even.



Pick Up

A game for 2 players

You need 6 stones numbered

(1) (2) (3) (4) (5) (6)

Each player takes it in turn to pick a stone. When each have 3 then each adds the numbers on his stones. The player with an odd total is the winner.

2nd Version

Play exactly as before but this time the winner is the player who has an even total.

INVESTIGATION

- (1) Can you work out how the first player should pick so that he wins?
- (2) Why can't you play this game with 8 stones (1 to 8)?
- (3) Will 10 stones (1 to 10) work? Try playing the game with 10 stones. It is a difficult game to investigate but fun to play.
- (4) What happens if we play with 5 stones numbered 1 to 5? Does this destroy the first players advantage?

6 CROSSING THE RIVER

You need the numbered stones for this puzzle.



Puzzle 1

Start with stones ① ② ③ on the left bank. Each of the stones is a man who wants to cross the river. Each man's weight is given by his number. ② weighs 2 units, ③ weighs 3 units, ① only weighs 1 unit. The boat can only carry a total weight of 3 units. If more than this is put in it sinks!

How will the 3 men cross the river?

Can you do it in 5 journeys?

Remember that someone has to row the boat back across the river to fetch his friend.

Example

First journey



① and ② cross

Second journey

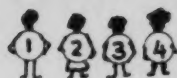


② stays on the right bank

① goes back.

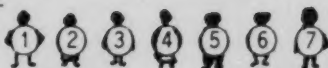
What next?

Puzzle 2



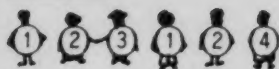
all want to cross the river.
This time the boat is bigger.
It can carry a total of 4
units.
Can you get everyone across
the river in 7 journeys?

Puzzle 3



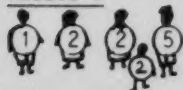
want to cross. The boat holds
7 units. can you do it in 9
journeys?

Puzzle 4



want to cross. The boat holds
4 units. Can you do it 7 journeys?

Puzzle 5



all want to cross. The boat holds 5 units.
Can you do it 5 journeys?

Puzzle 6



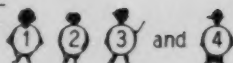
want to cross. The boat can take
any two stones at a time but
no two consecutive numbers can
travel together (eg. (1) cannot travel
with (2), (3) cannot travel with
(2) or (4). Can you get all across
in 5 journeys?

Puzzle 7



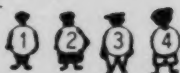
and (6) want to cross. The boat
holds 3 stones. Once again no
consecutive numbers can travel together.
Can you do it in 5 journeys.

Puzzle 8



wish to cross. The boat holds two stones only. No consecutive numbers can travel together or be left together on the bank while the boat crosses. Can you do this in 5 journeys?

Puzzle 9



and 5 wish to cross. The boat holds 2 stones and once again no consecutive numbers can travel together or be left together on the bank. What is the least number of journeys needed for all to cross?

Puzzle 10



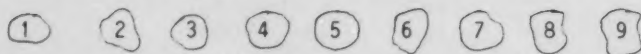
All these people are waiting to cross. The boat holds a weight of 5 units. What is the largest total weight that can cross over in 5 journeys?

INVESTIGATION

- (1) What is the greatest total weight that can cross over the river if the boat holds 3 units and you are allowed 5 journeys?
- (2) Can you find a general rule that will tell us the total weight that can cross for all sizes of boats and all numbers of journeys?

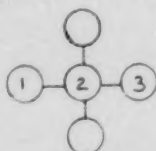
7 SLIDING PUZZLES

Take some small white stones.
Write a number on each one.



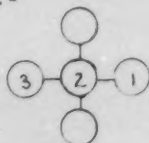
CROSS PUZZLE

Take the stones numbered 1, 2, 3 and place them on the cross in this position:



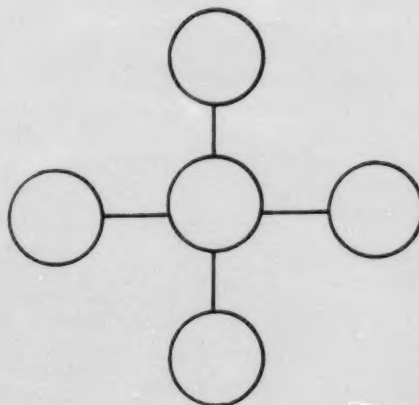
Now in 8 moves

try and rearrange them to look like this:-



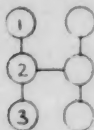
Each move

is made by moving a stone along a line to an empty circle.

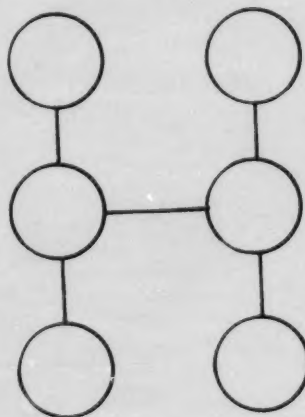
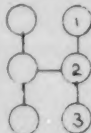


H puzzle

Start with the stones like this:

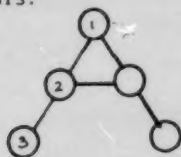


In 13 moves
make this

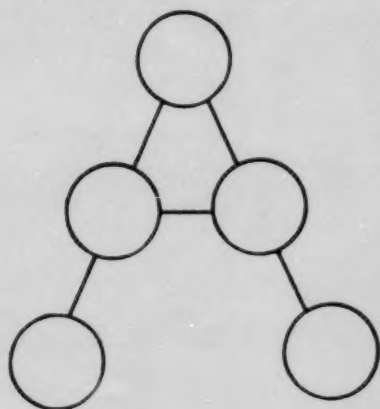
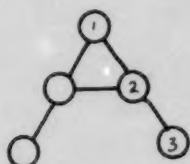


A puzzle

Start with the stones
like this:

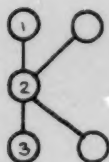


Now try and slide the stones
so that they finish like
this:

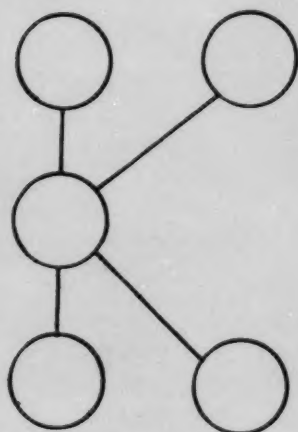
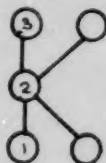


K Puzzle

Start with the stones like this

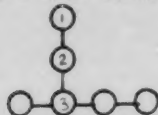


and make

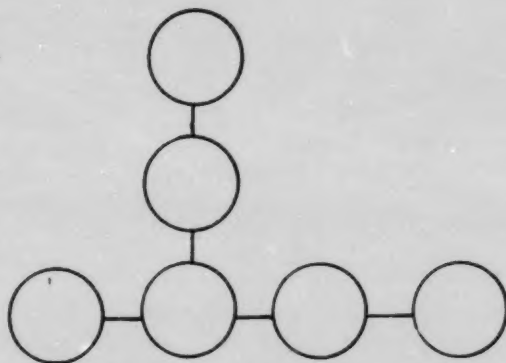
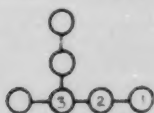


L Puzzle

Start with stones like this:-

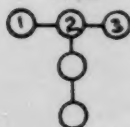


and finish like this:

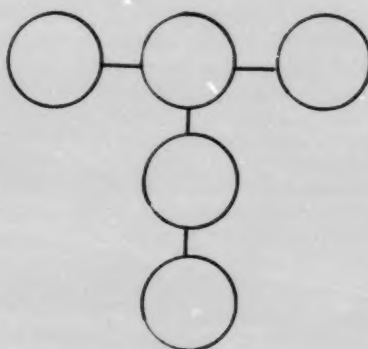
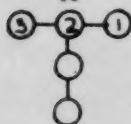


T Puzzle

From

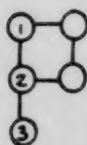


to

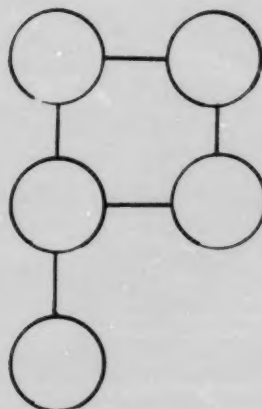
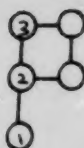


P Puzzle

From

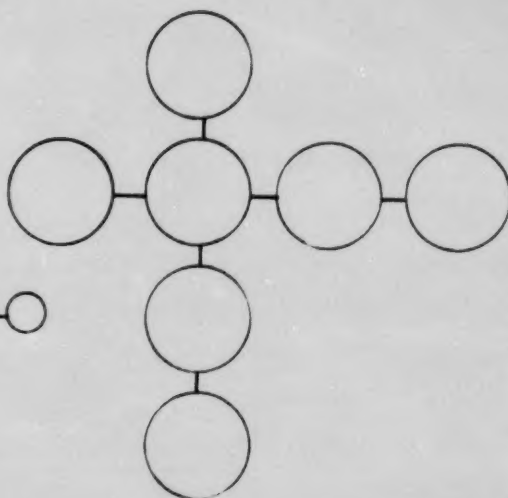
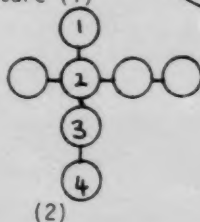
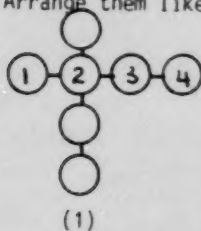


to



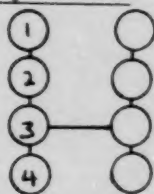
Big Cross Puzzle

Take stones numbered 1, 2, 3, 4
Arrange them like picture (1)

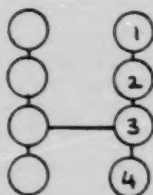


Move them to make picture (2)
Can you do it in 24 moves?

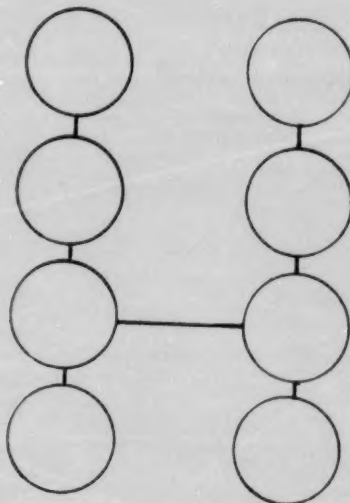
Big H Puzzle



Start



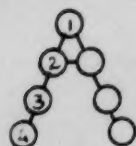
Finish



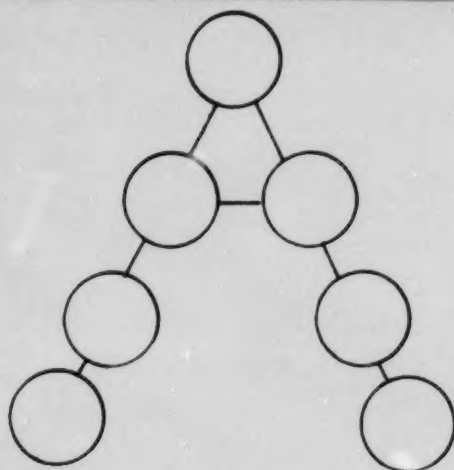
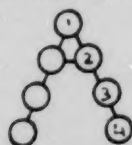
How many moves does it take
to go from start to finish position?

Big A Puzzle

Start with this

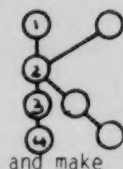


and make this

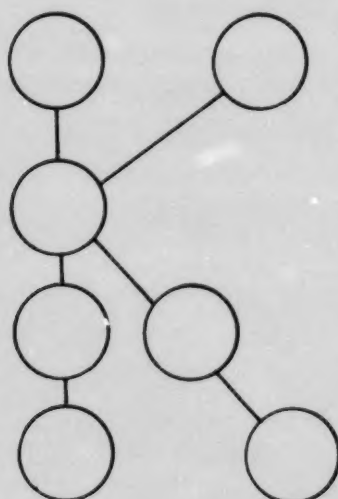
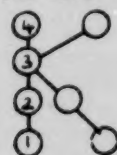


Big K Puzzle

Start with

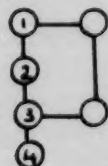


and make

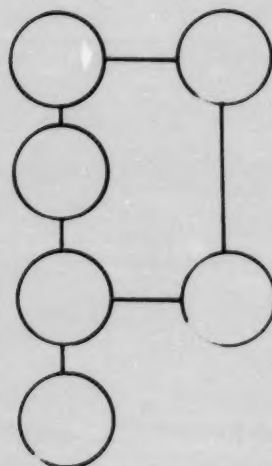


Big P Puzzle

From

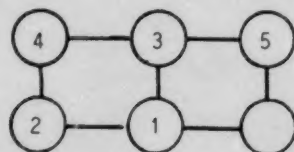


to

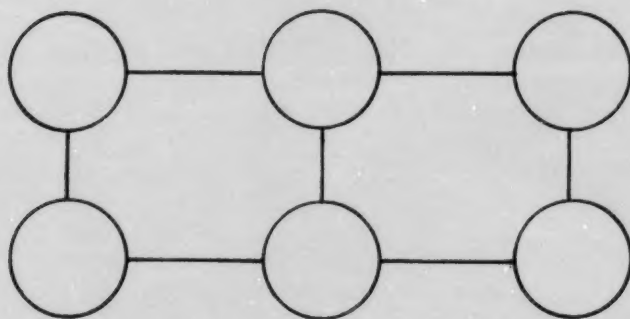
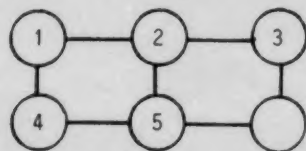


Sliding Numbers (1)

Place the numbered stones on the grid like this:

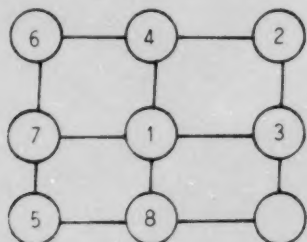


Now by sliding the stones into the empty space can you rearrange the stones in order to give:

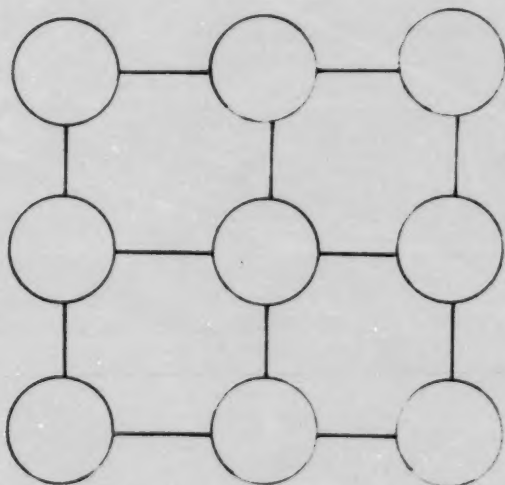
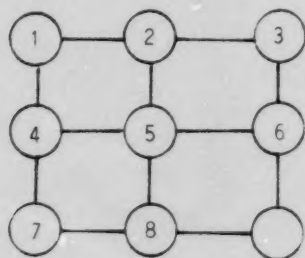


Sliding Numbers (2)

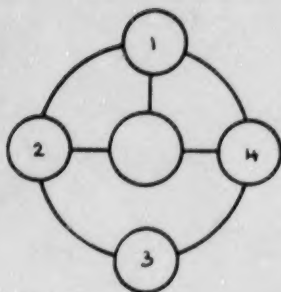
This time you will need 8 stones numbered 1 to 8.
Place them on the grid like this:



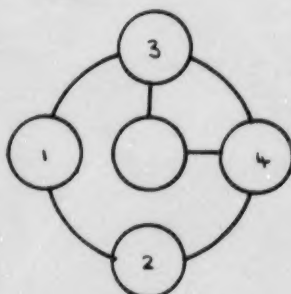
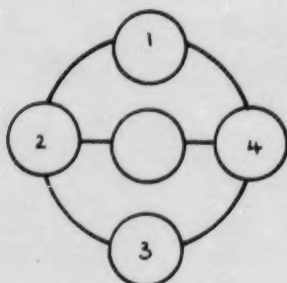
Now by sliding stones into empty spaces can you arrange the stones in order:-



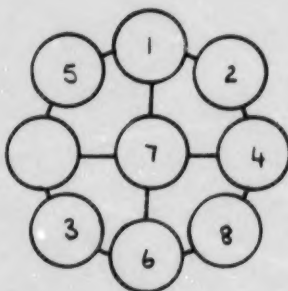
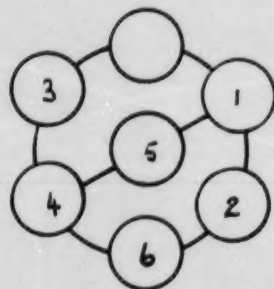
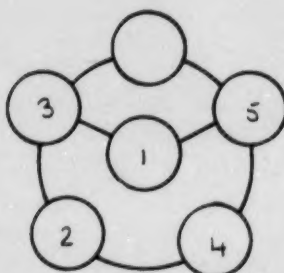
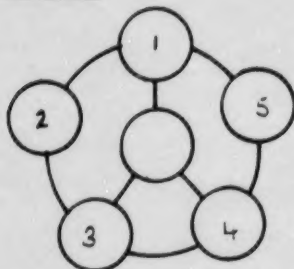
Sliding Circles



Start with the numbered stones as shown in the circles. Then by sliding along to the empty circle try and rearrange the numbers in order clockwise around the ring.

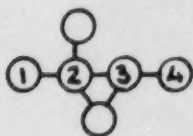


Harder ones.

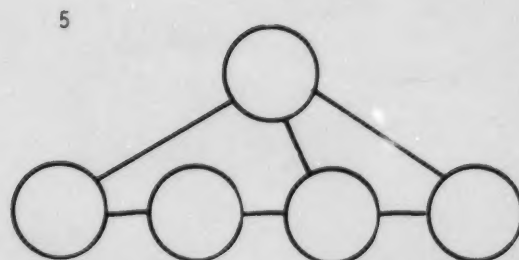
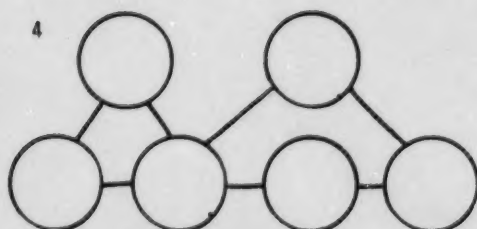
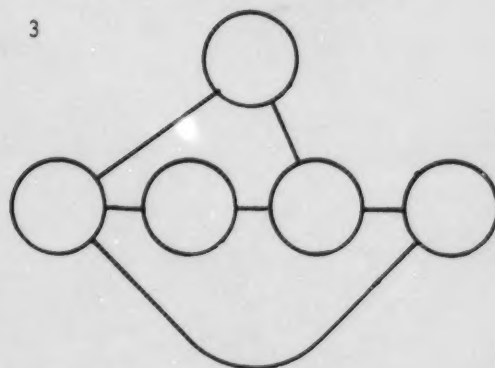
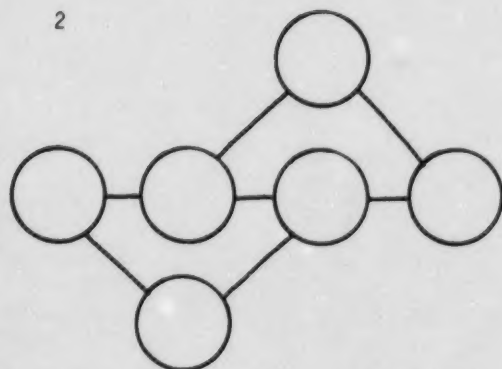
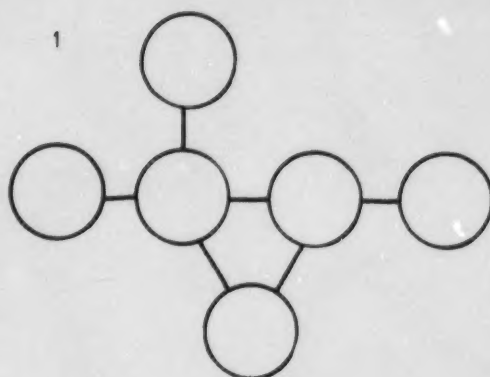
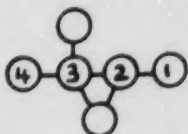


4 Stones in a line

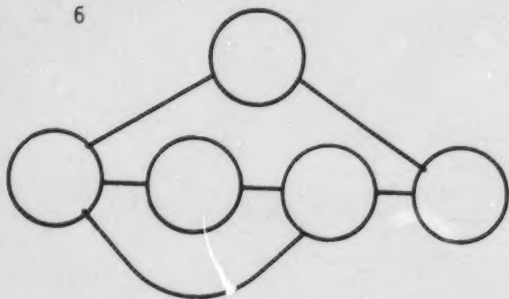
The puzzles that follow are all of the same kind. Place four numbered stones in the row of four circles.



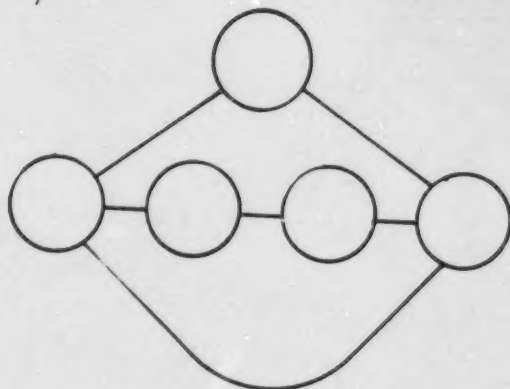
In as few moves as possible try and reverse the order.



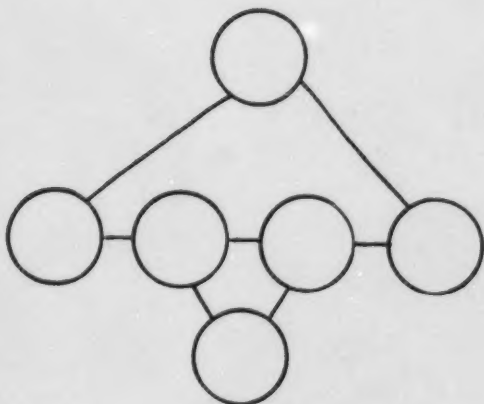
6



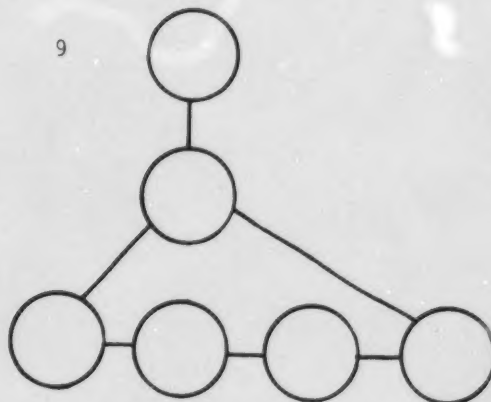
7



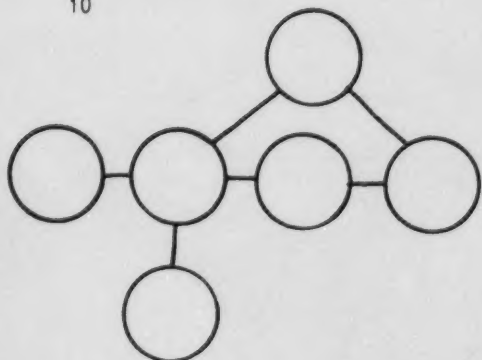
8



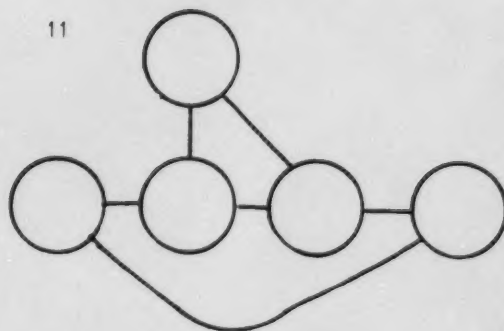
9



10

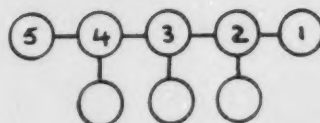
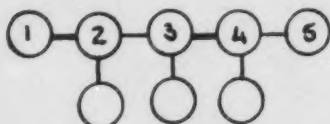


11

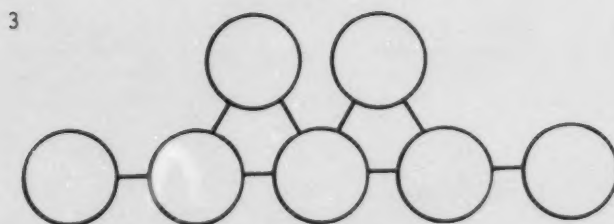
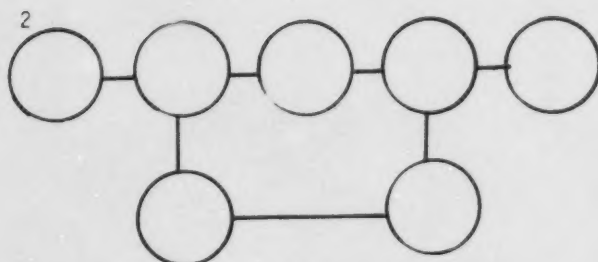
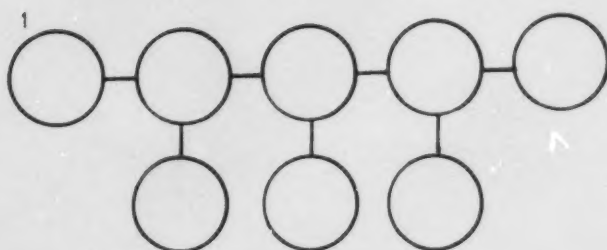


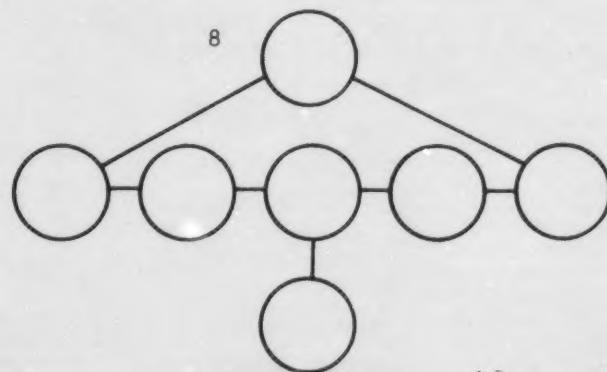
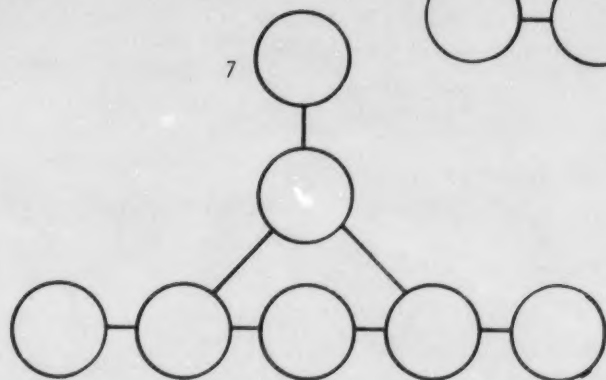
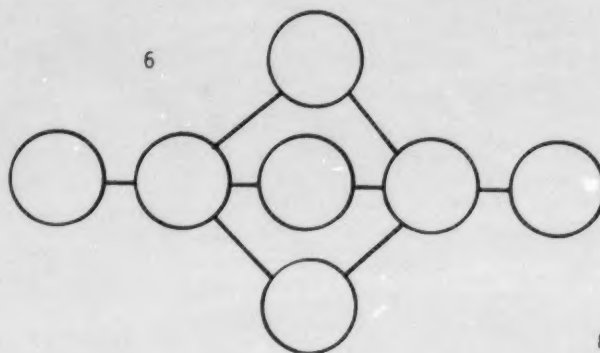
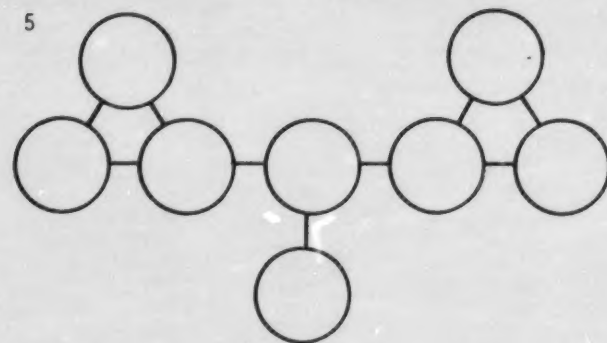
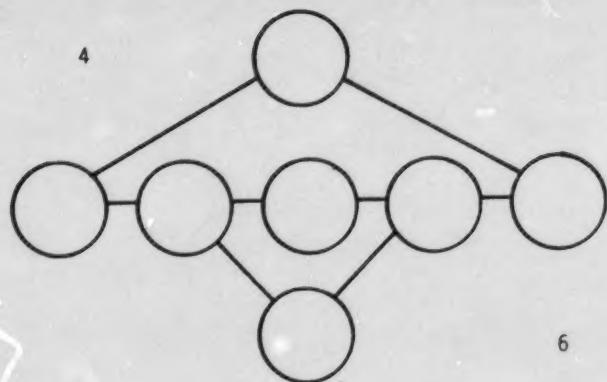
5 Stones in a line

In these puzzles place 5
numbered stones in a row in the
5 circles.

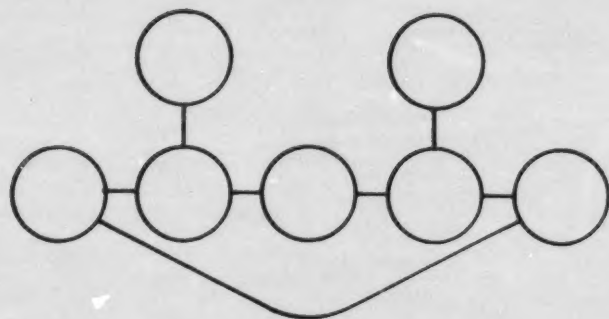


In as few moves as possible
try and reverse the order.

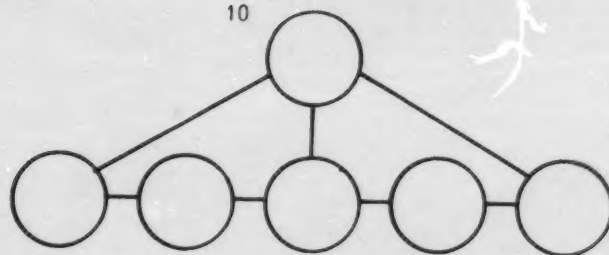




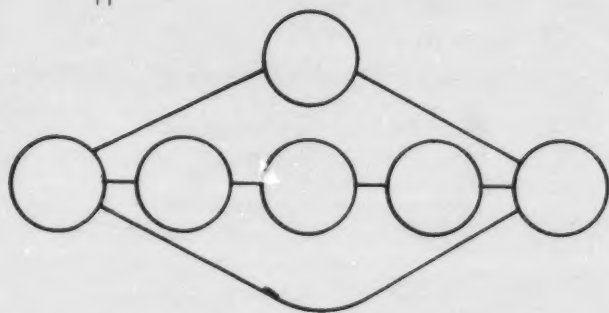
9



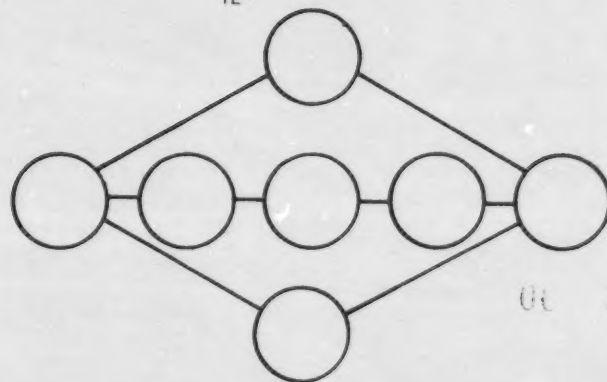
10



11



12

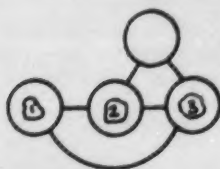


00 44

38

43

INVESTIGATION



- (1) Use this diagram and stones (1) (2) and (3) placed in a row. Is it possible to slide the stones so that all possible arrangements of (1) (2) and (3) are made?
- (2) Can you find a system of moves that brings all 6 arrangements into line one after the other?

8 SOLITAIRE PUZZLES

In these puzzles place a stone or shell in each circle except for that marked E (empty). On each go jump one stone over another into an empty circle. Take away the stone you jumped over. The aim is always to end with just one stone.

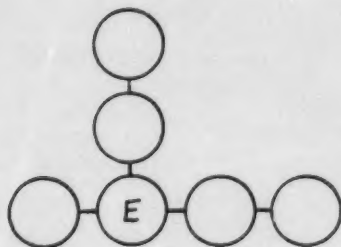
- (1) 5 stone solitaire



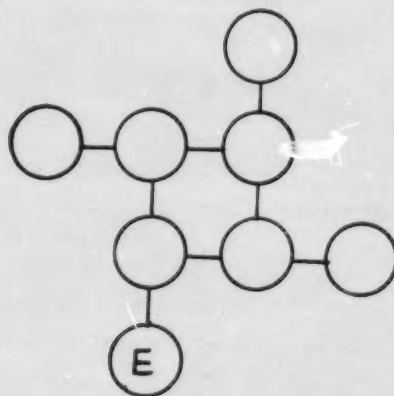
- (2) 7 stone solitaire

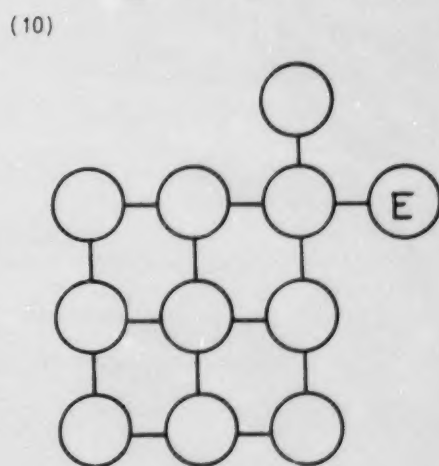
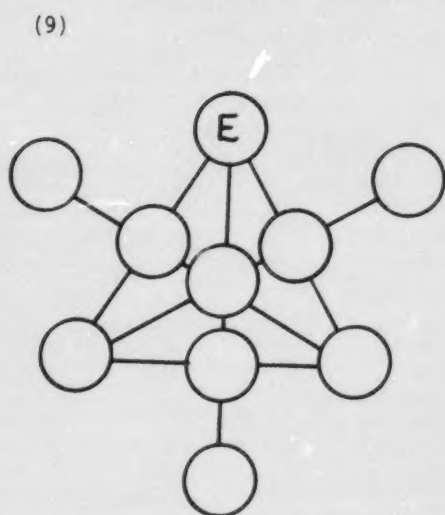
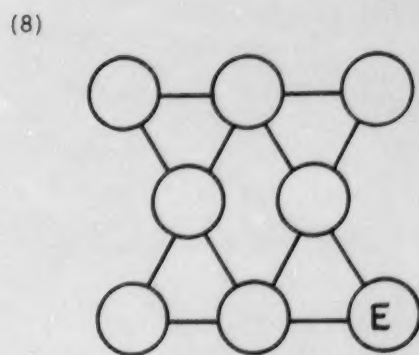
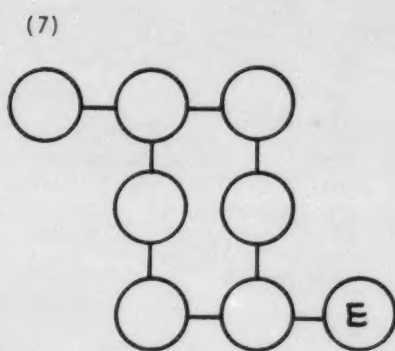
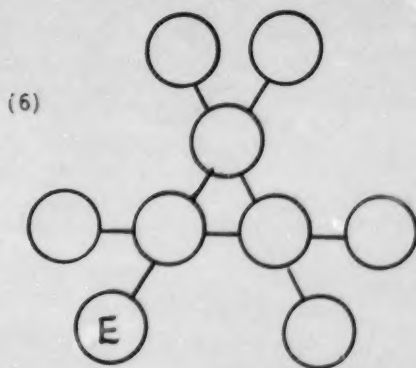
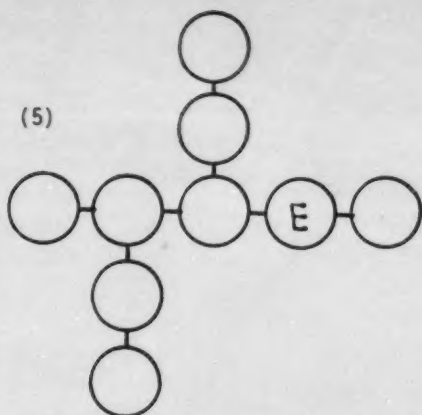


- (3)

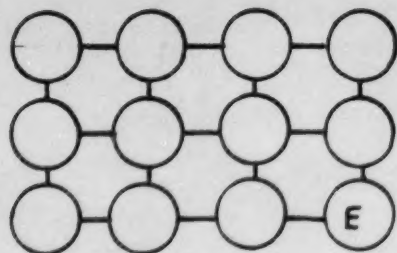


- (4)

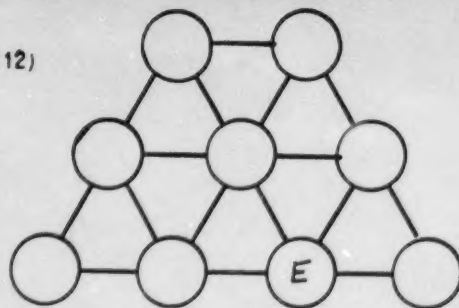




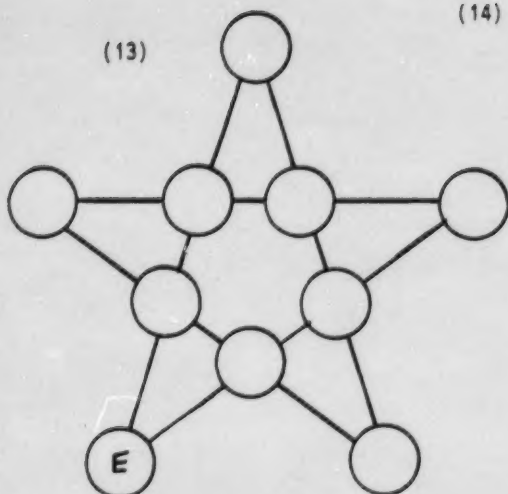
(11)



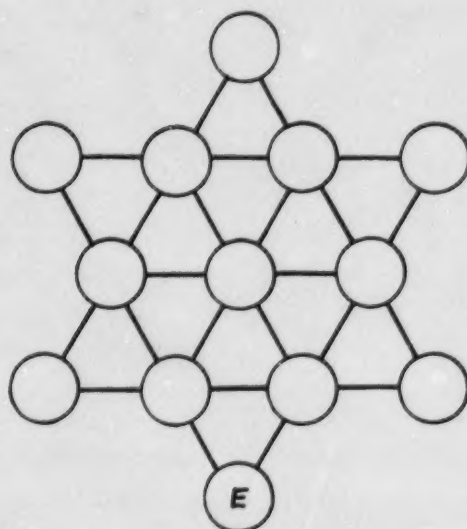
(12)



(13)



(14)



INVESTIGATION

- (1) In 5 stone solitaire, the 1st puzzle in this section, is it possible to start with the empty circle in a different position?
- (2) What happens in 6 stone solitaire (7 circles in a row, one empty)? Can you work out which simple straight row solitaire puzzles will be possible and which impossible?

9 MAGIC SHAPES

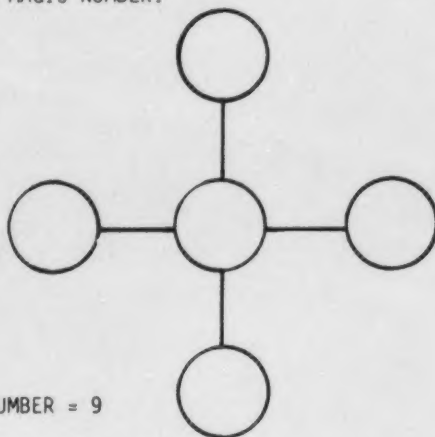
For these puzzles you will need stones numbered from 1 to 9.
The aim of all these puzzles is to make rows of numbers which add to the same total, which we call the MAGIC NUMBER.

Magic Cross

- (1) Use the 5 stones.

(1) (2) (3) (4) (5)

Place one in each circle so that both rows add to 9.



MAGIC NUMBER = 9

- (2) Use the same stones

(1) (2) (3) (4) (5)

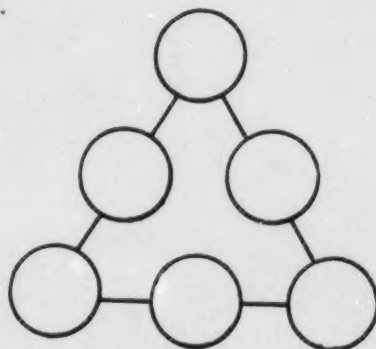
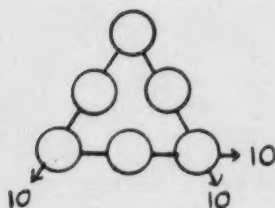
Make the Magic Number equal to 10.

- (3) Use the same stones;
make the Magic number equal to 8.

- (4) Using the stones (2) (3) (4) (5) (6) can you find three different Magic Numbers for the Magic Cross.

Magic Triangle

- (1) Use stones (1) (2) (3) (4) (5) (6).
Put one in each circle
So that the Magic number
for each side is 10.



- (2) Use stones
(1) (2) (3) (4) (5) (6)
Make the Magic number
in the triangle 11.

- (3) Use stones
(1) (2) (3) (4) (5) (6)
Make the Magic number
12.

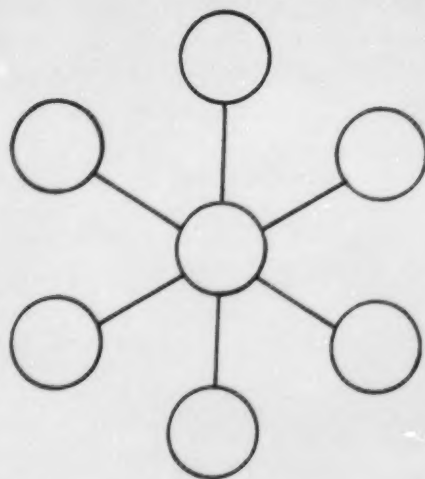
- (4) Using the same stones can you find one other
Magic number?

Magic Star

(1) Use the stones

(1) (2) (3) (4) (5) (6) (7).

Put one in each circle
so that the Magic number
is 12.



(2) Use the same stones.

(1) (2) (3) (6) (5) (6) (7)

Make the Magic Number
10.

(3) Use the same stones

(1) (2) (3) (4) (5) (6) (7)

Make the Magic Number 14.

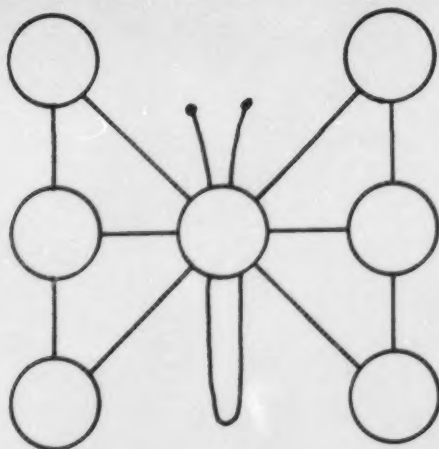
(4) Now use the stones numbered (3) (4) (5) (6) (7) (8) (9)
on the Magic Star. What 3 Magic numbers
can you make?

Magic Butterfly

Use the stones

(1) (2) (3) (4) (5) (6) (7)

Put one in each circle
so that the total of each row,
the Magic Number, is 12.



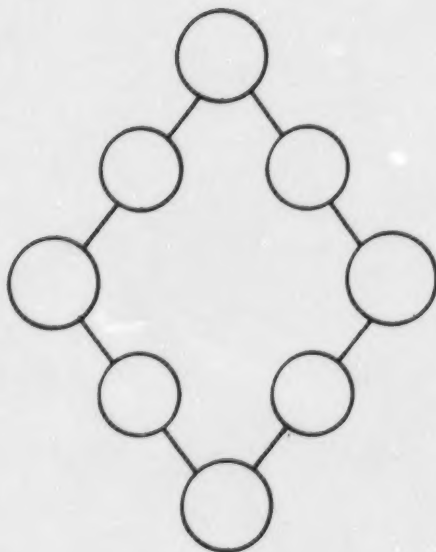
Magic Diamond

This is quite a hard puzzle.

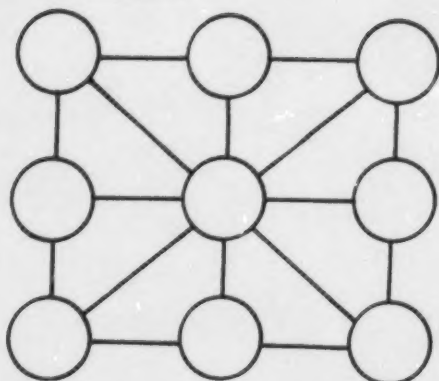
Use the stones

(1) (2) (3) (4) (5) (6) (7) (8)

Put one in each circle so that
the Magic Number is 12.



Magic Square

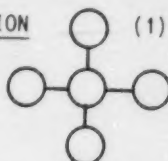


Use the stones numbered
1 to 9.

(1) (2) (3) (4) (5) (6) (7) (8) (9)

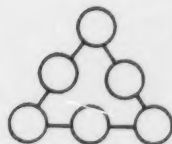
Put one stone in each
square so that the Magic number
for each line is 15.

INVESTIGATION



(1)

In the Magic Cross we found 3 magic
numbers. For any set of 5 consecutive
numbers, which should be put at the
centre of the cross to make it magic?



(2)

In the Magic triangle we found
4 magic numbers. For any set of 6
consecutive numbers how should
they be placed, what numbers
should go at the corners, to make
the triangle magic?

10 NUMBER PUZZLES

Take 4 stones numbered 1, 2, 3 and 4
Choose which ones you need
to put in the circles to make the
equations true. For example in
the first equation.

$$\bigcirc + \bigcirc + \bigcirc = 6$$

we could put

$$\textcircled{1} + \textcircled{2} + \textcircled{3} = 6$$

This makes the equation true.
Now try and make the other equations true.

$$\bigcirc + \bigcirc + \bigcirc = 6$$

$$\bigcirc + \bigcirc + \bigcirc = 8$$

$$\bigcirc + \bigcirc + \bigcirc = 7$$

$$\bigcirc + \bigcirc + \bigcirc = 9$$

$$\bigcirc + \bigcirc - \bigcirc = 4$$

$$\bigcirc + \bigcirc - \bigcirc = 1$$

$$\bigcirc + \bigcirc - \bigcirc = 5$$

$$\bigcirc + \bigcirc - \bigcirc = 3$$

$$\bigcirc \times \bigcirc + \bigcirc = 6$$

$$\bigcirc \times \bigcirc + \bigcirc = 7$$

$$\bigcirc \times \bigcirc + \bigcirc = 13$$

$$\bigcirc \times \bigcirc + \bigcirc = 11$$

$$\bigcirc \times \bigcirc - \bigcirc = 5$$

$$\bigcirc \times \bigcirc - \bigcirc = 10$$

$$\bigcirc \times \bigcirc - \bigcirc = 7$$

$$\bigcirc \times \bigcirc - \bigcirc = 2$$

INVESTIGATION

- (1) Take a different set of numbered stones, 5, 6, 7 and 8. How many different answers can be made if the stones are placed in

$$\bigcirc + \bigcirc + \bigcirc =$$

- (2) Using the same stones, how many different answers can be made if they are put in the sentence

$$\bigcirc + \bigcirc - \bigcirc =$$

- (3) Use any 4 consecutive numbers, a , $a + 1$, $a + 2$, $a + 3$ and the sentence:

$$\bigcirc + \bigcirc - \bigcirc =$$

What would be the smallest possible answer?

What would be the largest possible answer?

Adding Puzzle (A)

Take the 4 stones marked 1, 2, 3 and 4. Put them in the circles so that they make a sum which adds up to 46. For example

$$\begin{array}{cc} \textcircled{4} & \textcircled{2} \\ + & \\ \textcircled{3} & \textcircled{1} \\ \hline \end{array}$$

adds to 73, So this is not the right answer.

$$\begin{array}{cc} \bigcirc & \bigcirc \\ + & \\ \bigcirc & \bigcirc \\ \hline \end{array}$$

Now try these. Can you make the answer to the sum

- a) 37 b) 64 c) 55

Subtraction Puzzle (A)

Using the same 4 stones can you make subtraction sums so that the answers are:-

- a) 11 b) 22 c) 31
d) 13 e) 29 f) 9

$$\begin{array}{cc} \bigcirc & \bigcirc \\ - & \\ \bigcirc & \bigcirc \\ \hline \end{array}$$

Adding Puzzle (B)

Take the stones numbered 1 to 6.
Place them in the circles to give
two 3 figure numbers. For
example

$$\begin{array}{r} \textcircled{2} \quad \textcircled{3} \quad \textcircled{5} \\ + \quad \textcircled{1} \quad \textcircled{4} \quad \textcircled{6} \\ \hline \end{array}$$

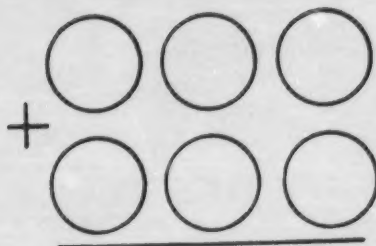
The answer to this sum would be

3 8 1

Can you arrange the stones so the
answer would be 579?

Now try and make these answers:

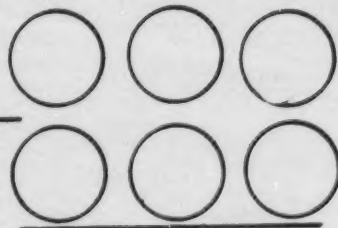
- | | |
|--------|--------|
| a) 777 | b) 399 |
| c) 489 | d) 993 |



Subtraction Puzzle (B)

Using the same 6 stones can you
make subtraction problems where the answers are

- | | | |
|--------|--------|--------|
| a) 333 | b) 531 | c) 111 |
| d) 107 | e) 188 | f) 276 |

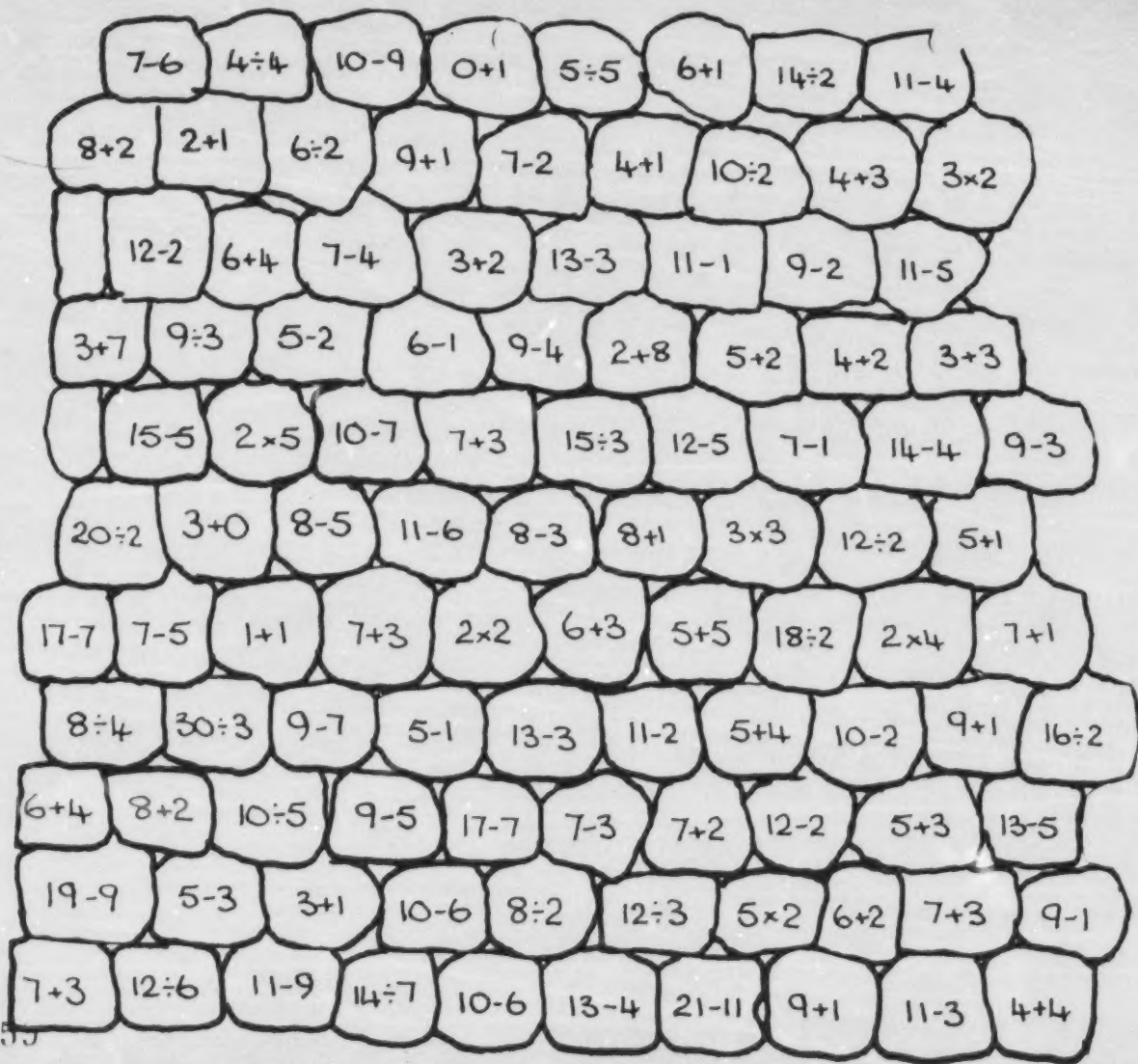


The Stone Wall Puzzle

(1) Take small stones and put them on the wall wherever the answer is 2. What do you notice?

(2) Clear off the stones. Now put the stones on all the sums that give the answer 3.

(3) Put stones where ever the answer is 4. Repeat with all numbers up to 9.

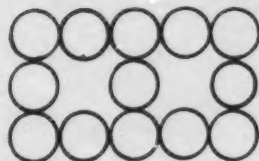


11 MIXED PUZZLES

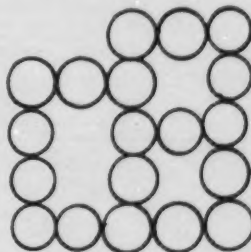
Path Puzzles

The aim of these puzzles is to lay a continuous line of stones in the circles so that all circles are filled. No jumping is allowed. Paths must go from circle to touching circle.

(1)

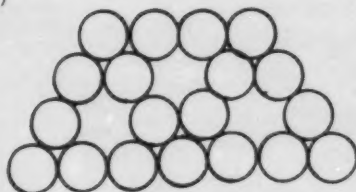


(2)

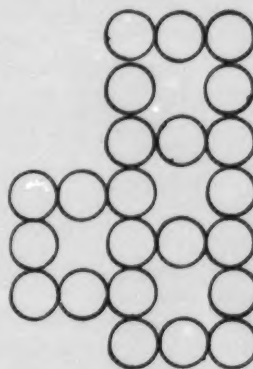


If you are doing these with a class of children it is good idea to tell them to put a stone marked S where they started the path and a stone marked F where they finished. Then you can quickly check if their answers are correct.

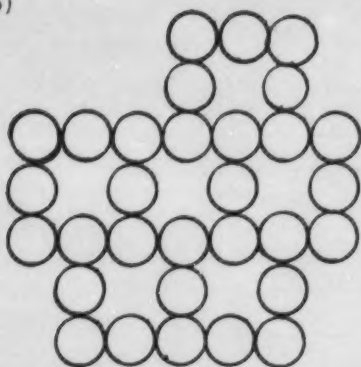
(3)



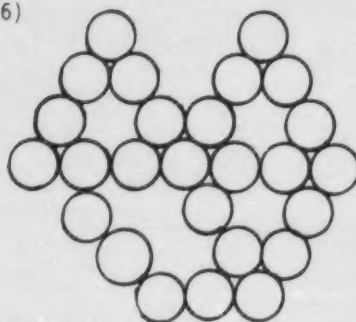
(4)



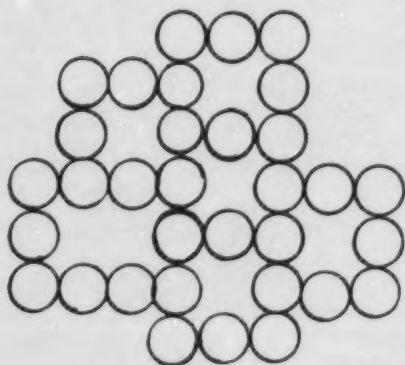
(5)



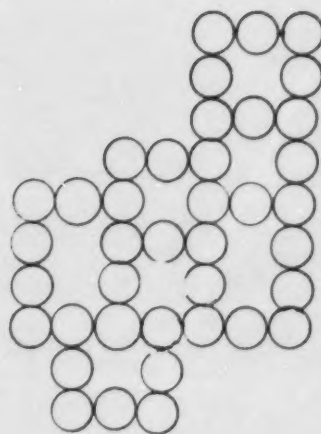
(6)



(7)



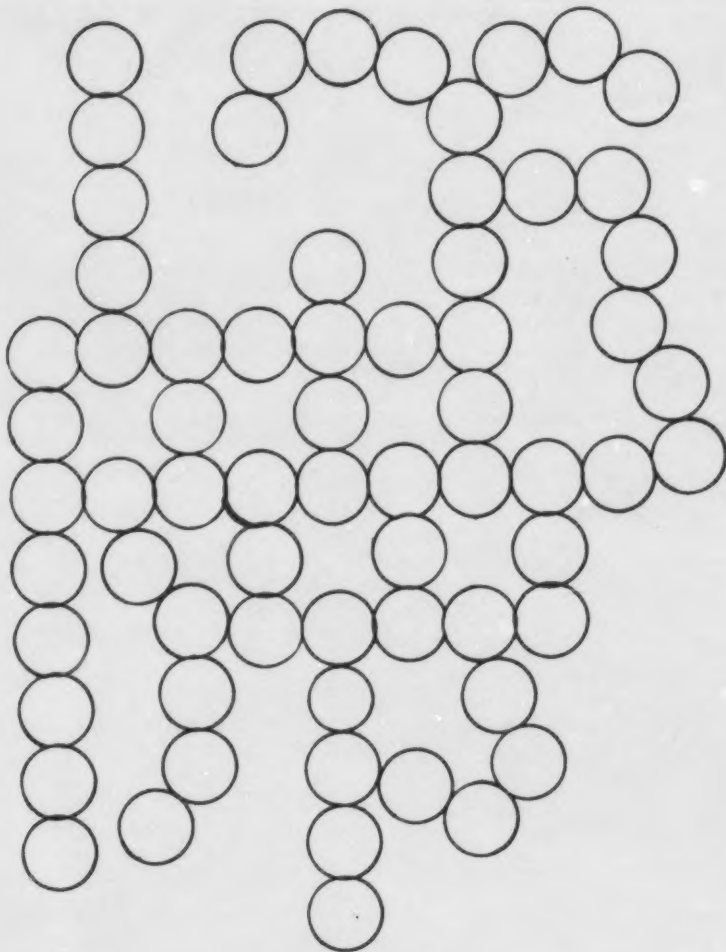
(8)



You may prefer to try these puzzles in reverse.
Cover each circle with a stone. Then pick up one at a time,
always going onto a touching circle. Can you pick up all
the stones?

Pathways game - for 2 players

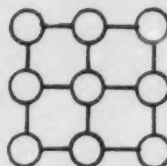
Each player has a pile of stones. The first player may put his first stone on any circle. The next player must put his stone in a circle touching this one. Play continues in this way until one player cannot go. He loses.



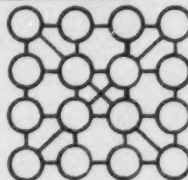
STONES IN A LINE

One in a line

- (1) Take 3 stones. Put them in the circles so that every row and column has just 1 stone in it.

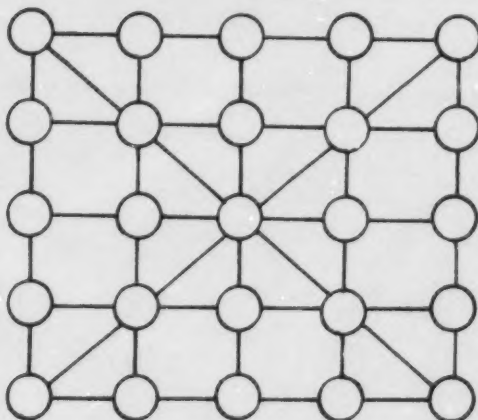


- (2) Take 4 stones. Put them so that there is just one in each row, column and diagonal.



- (3) Take 8 stones. Place them one stone per circle so that there are 2 in each row, column and diagonal.

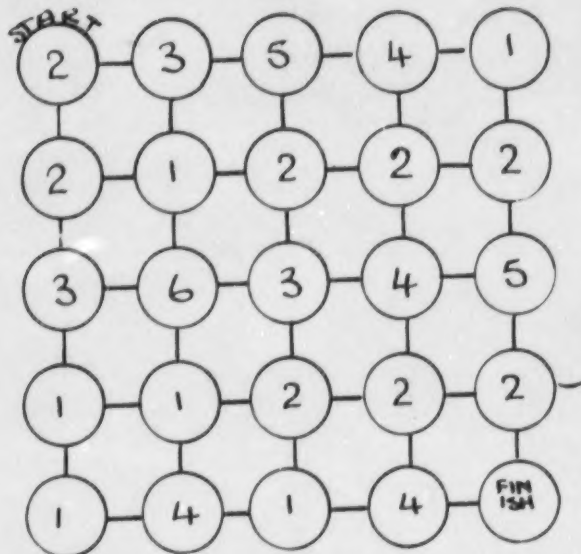
- (4) Take 12 stones. Place them one per circle so that there are 3 in each row, column and diagonal.



- (5) Place 5 stones, 1 in each line.
- (6) Place 12 stones so there are 2 in each line.
- (7) Place 15 stones so that there are 3 in each line.
- (8) Place 20 stones so that there are 4 in each line.

STEPPING STONES

PUZZLE: Place your stone on start. Move the number shown (2 steps). You can go in any direction; you could go down to 3, across to 5 or one step down and 1 step across to 1. Each time you end a move look at the number you land on. Move on again that number of steps. Try and reach 'finish' in the fewest possible moves.



START

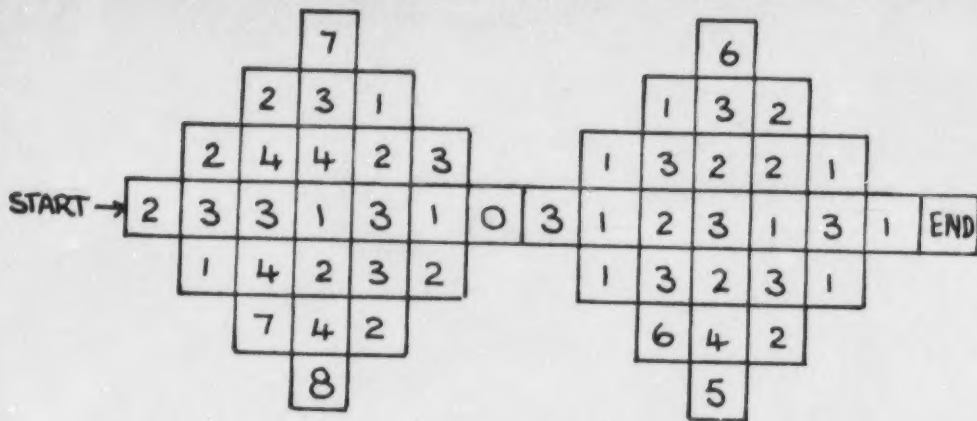
2	5	3	3	2	1	1	2	3	1	0	2	1	3	1	2
1	1	2	3	1	2	3	1	2	2	0	1	3	2	1	3
1	3	6	1	4	3	1	1	1	1	0	3	2	1	2	1

GAME:

Players each place a stone at the start (2,1 or 1). The number of steps you take is given by the number on which your stone lies. Take it in turn and race to the finish. First there is the winner.

FINISH

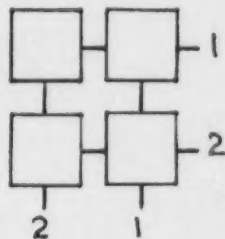
1	1	3	2	3	2	1	4	0	1	1	2	4	2
1	1	1	2	3	1	2	5	0	1	2	1	3	1
1	2	1	2	2	3	1	2	0	1	3	1	1	4



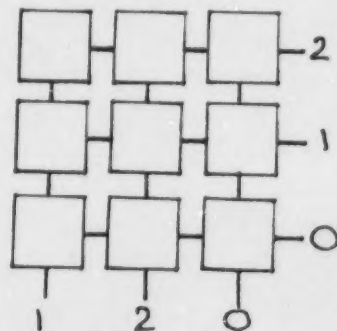
(2) ROWS AND COLUMNS

You are allowed to put one stone in each square. Can you put stones so that there are the number shown in each row and column?

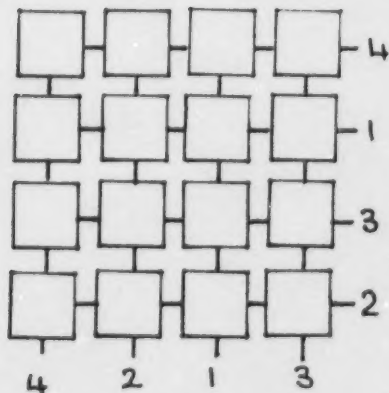
(a)



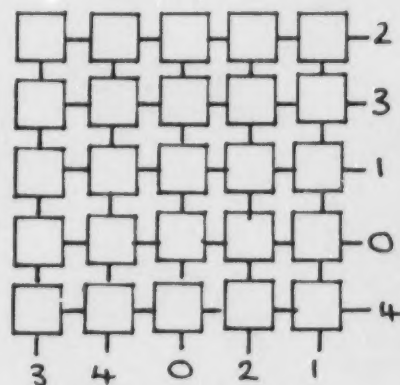
(b)



(c)

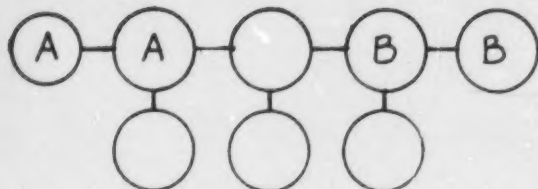


(d)



EXCHANGE PUZZLES

1.

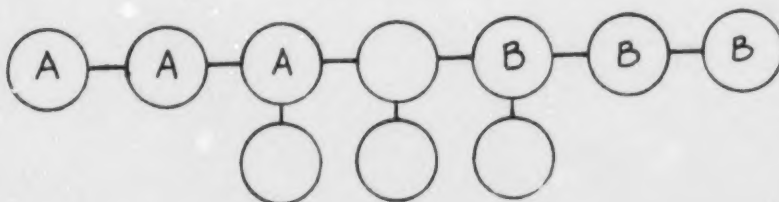


Put stones on the circles marked with an A.

Put shells on the circles marked with a B.

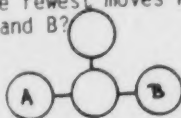
Can you move the stones to B and the shells to A? (No jumping)

2. Shells at A. Stones at B. Now try and exchange (Fewest possible moves = 38)



INVESTIGATION

(1) What are the fewest moves needed to exchange A and B?

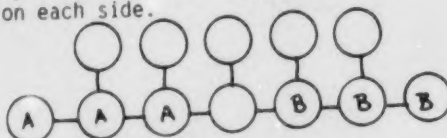


(2) Now lengthen each side



What are the fewest moves needed to exchange A's and B's?

(3) Try and predict what will happen with 3 on each side.



Can you find a relationship between the number of stones and the number of moves?

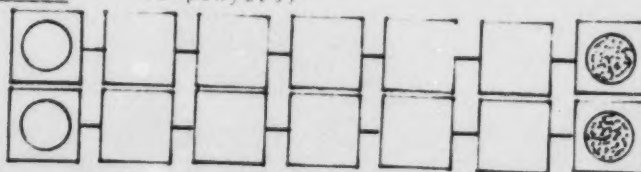
12 SIMPLE GAMES

(1) MOSQUITO (2 players)



Place a stone or shell on each circle. Each player takes 1, 2 or 3 stones starting from the left. The player who takes the stone covering the mosquito is bitten and loses.

(2) TRAPPING (2 players)

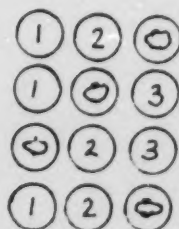


Each piece can move any number of steps right or left. No jumping. Play in turn. The winner is the player who blocks the other player's pieces so they cannot move.

(3) PLUS TEN (2 players)



Each player takes it in turn to move and place the stone on a circle. The numbers where the stone is placed are added. The player scoring 10 or more loses.



EXAMPLE

TOTAL

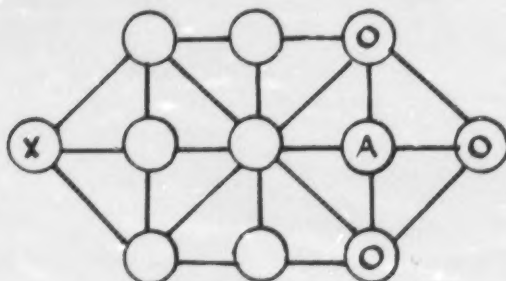
A	3
B	5
A	6
B	9

Player A loses.

4.

PIGS AND DOGS

Place one shell for a pig at X and 3 stones for dogs on the O's. The aim is for the player moving the dogs to trap the pig so it cannot move. Dogs can move from left to right, the pig can move in any direction. Move one piece in turn. If the pig gets past the dogs, it has won.

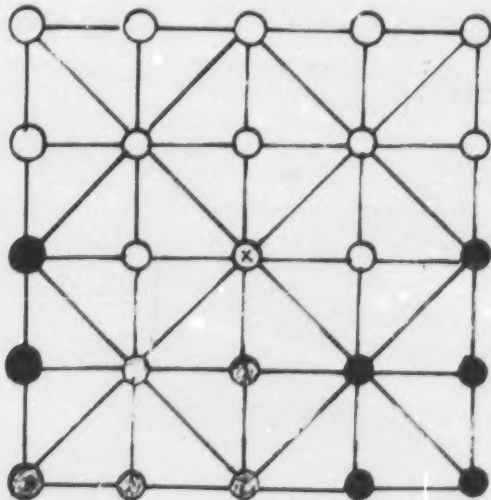


Alternative version: Pig starts at A. Dogs are placed in any position, one per go. Then as before.

5.

CATCH THE PIG

The pig starts at X. The dogs start on the dark circles. All pieces can move in any direction. The pig can kill dogs by jumping over them. The dogs aim to trap the pig so it cannot move. The pig aims to kill all the dogs.



6. Place Value Game

This is a good game to play with a class. Each child needs stones numbered from 0 to 9. The teacher also has a set which he puts in a cup or tin. Each child also has place value chart.

0 1 2 3 4 5 6 7 8 9

H	T	U

The teacher picks at random one stone from his cup. He tells the children which it is. They all have to place that number in one column of their chart. Once placed it must not be moved.

The teacher picks another and calls its name. The children place it. The teacher does this until the chart is full. The aim of the game is for the children to try and get the highest possible number.

Alternative versions

Once the children have got the idea of this game try these variations:

- (1) The aim is to get the smallest number.
- (2) The aim is to get the largest even number.
- (3) The aim is to get the smallest odd number.
- (4) Change the column values to include decimals.

7. Spoof. For 2 or more players

Each player has 3 stones. The aim of the game is for each player to try and guess the total number of stones the players are hiding in their hands.

Each player puts his hands behind his back and decides how many stones to hide in his right hand; one, two, three or none. All the right hands are then held out in front. Each player guesses in turn the total number hidden. When all have guessed the hands are opened. The player who guessed correctly, or was nearest, wins.

Each player starts with a pile of 5 shells(or stones). Each time he looses he must pay one shell to the winner. The game continues until one player has no shells left.

8. Odds and Evens

A game for 2 players

Each player has 3 stones. He puts his hands behind his back and hides 1, 2, 3 or no stones in his right hand. Then both players hold out their right hands. One player guesses whether the total number of stones is odd or even. The hands are opened. If the guess was right, he takes one stone from the other player. If he is wrong he gives one stone.

On the next go the other player is the one to guess. The game continues until one player has lost all his stones.

SOLOMON ISLAND GAMES

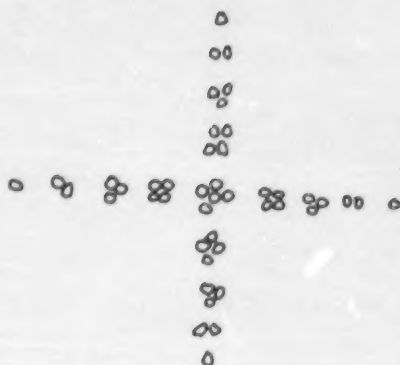
(1) PINO PINO

Layout the stones or shells in the pattern shown. The 'player' turns his back so he cannot see the pattern. Another person kneels by the stones ready to make stones as he is instructed by the 'player'. The 'player' says:

'Take one from the first pile, leaving none!

'Take one from the next pile, leaving stone !

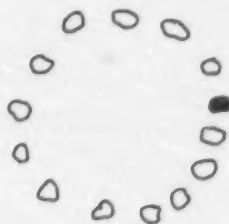
As he gives instructions the stones are taken. First he works along one arm of the pattern. Then he starts on the second arm. The aim is correct instructions until no stones are left.



(2) BANGARA

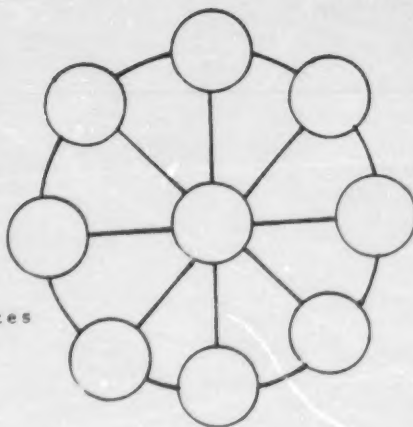
Any number of stones or shells are placed in a ring. One must be distinctive and is called the king.

The first player starts counting at any point in the ring counting from 1 to 10 and touching stones. The tenth object is removed from the ring and the next player counts on from the next object. Play continues until one player finishes his count on the king. He has then lost.



(3) MOVE MOVE (Eight legs)

The playing board is as shown.
One player places 3 stones on the
top 3 circles, the other player
3 shells on the bottom 3 circles.
Each player moves a piece in turn
along a line to an empty space.
The first player to get his 3 pieces
in a line wins.



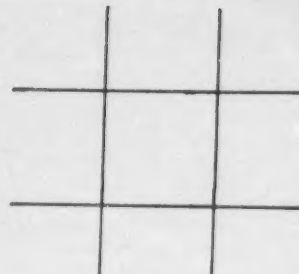
ALTERNATIVE VERSIONS

The board is empty at the start of
play. Each player places one of his
pieces in turn wherever he wants.
Then game continues as before.

INVESTIGATION

Make a Line Games

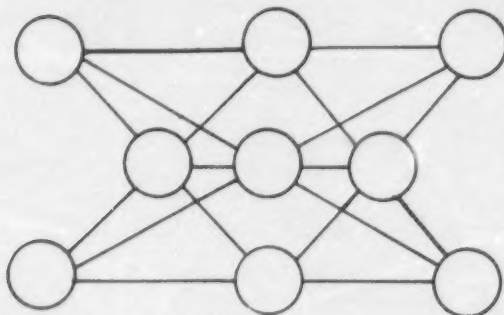
- (1) There are many different boards and rules for games of this kind. The most common version is Tic Tac Toe or Noughts and Crosses. One player has stones. The other has shells. Each puts one in turn in one of the boxes. The first to get 3 of his pieces in a line wins. How should this game finish?



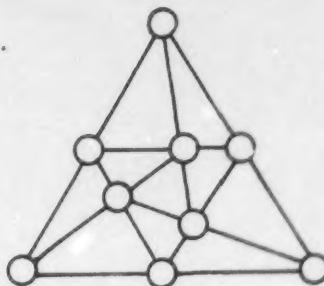
- (2) If we limit each player to 4 pieces we get a more interesting game. If, when both 8 players have placed all their pieces the game is drawn then continue by playing a piece in turn into an adjacent empty space. Now one player will win.

- (3) It is interesting to play the game on different boards. Here are a few suggestions. The first two attempt to make each place of equal importance by having an equal number of lines passing through every position. The third is a traditional version from Zimbabwe.

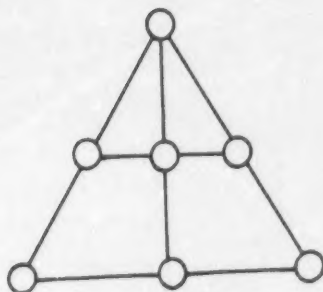
- (a) Every circle is on 3 lines



- (b) Every circle is on 3 lines.



- (c)



Now try making up your own board.

(4) Here are some other ways to vary the game

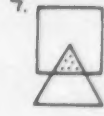
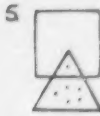
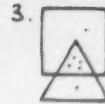
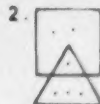
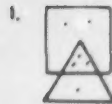
- Increase the size of the board or grid and make it a 4 in a line game.
- Vary the number of pieces each player has.
- Change the winning rule so that the winner is the player with most lines or so that the first to make a line loses.

Answers & Notes

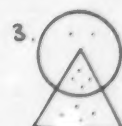
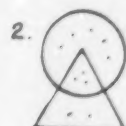
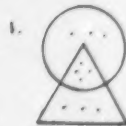
(Page 3)

Sum Puzzles

Answers



(Page 4) More sum puzzles



Notes on Investigation

There are 3 ways to solve these Sum puzzles

- (1) by trial and error using the stones.
- (2) by simple logic.
- (3) by simple algebra.

Using logic reason as follows:

- Step 1. 100 stones altogether, 67 in the circle
 $\therefore (100 - 67)$ must be outside the circle
 $\therefore 33$ must be outside the circle but inside the triangle.



- Step 2. 59 stones in the triangle.
 $\therefore (100 - 59)$ must be outside the triangle
 $\therefore 41$ must be in the circle outside the triangle.



- Step 3. 100 altogether
 $\therefore 100 - (41 + 33)$ in the intersection



Using algebra



Let the number of stones in each part
be a, b , and c as shown.
Then

- (1) $a + b + c = 100$ (100 stones altogether)
- (2) $a + b = 67$ (67 in circle)
- (3) $b + c = 59$ (59 in square)

Substitute (2) in (1)

$$\begin{aligned}(a + b) + c &= 100 \\ 67 + c &= 100 \\ \therefore c &= 33\end{aligned}$$

Substitute (3) in (1)

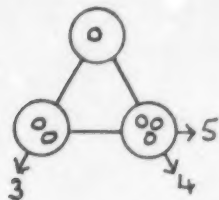
$$\begin{aligned}a + (b + c) &= 100 \\ a + (59) &= 100 \\ \therefore a &= 41\end{aligned}$$

Substitute $a = 41$ in (2)

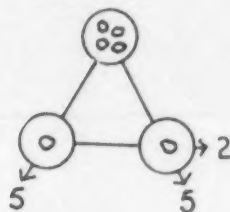
$$\begin{aligned}41 + b &= 67 \\ \therefore b &= 26\end{aligned}$$

So this simple puzzle can be used at almost any level and the kind of solution the children use can be suited to their ability to reason and use symbols.

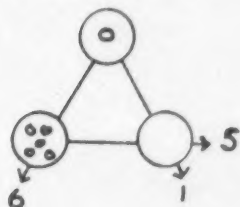
Puzzle 1



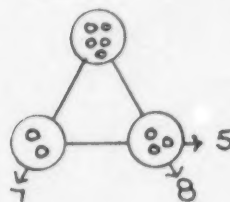
Puzzle 2



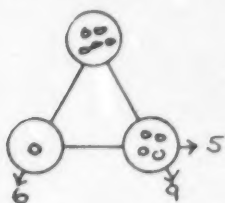
Puzzle 3



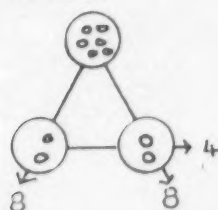
Puzzle 4



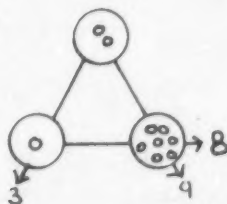
Puzzle 5



Puzzle 6



Puzzle 7



(Page 8) Triangle Puzzles - Notes on Investigation

- (1) If we have 10 stones then the 3 totals must sum to 20. Also no single total can be greater than 10.

With 8 stones the 3 totals must sum to 16.
No single total can be greater than 8.
Possible arrangements are:

8,8,0	8,7,1	8,6,2	8,5,3
8,4,4	7,7,2	7,6,3	7,5,4
6,5,5			

- (2) Again we can use logic or algebra to solve these puzzles.

10 stones

Using logic: The bottom row has 8 stones.

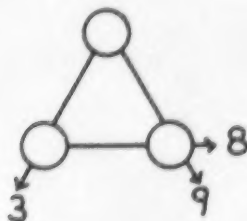
$\therefore$ must be $(10 - 8) = 2$ in top circle.

The left side has 3 stones.

$\therefore$ must be $(10 - 3) = 7$ in right circle.

The right side has 9 stones.

$\therefore$ must be $(10 - 9) = 1$ in left circle.



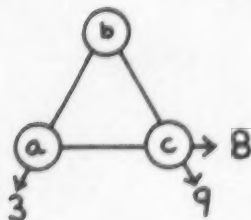
Using symbols

Make equations $a + b + c = 10$

$$a + b = 3$$

$$b + c = 9$$

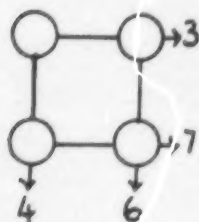
$$a + c = 8$$



Solve by substitution. Note that there are 4 equations and 3 unknowns.

- (3) Puzzles based on the square will in general not have unique solutions.

10 stones



If we make the equations for this example:

(1) $a + b + c + d = 10$

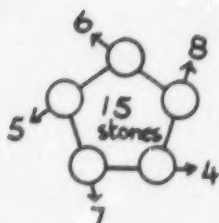
(2) $a + b = 3$

(3) $b + c = 6$

(4) $c + d = 7$

(5) $a + d = 4$

At first glance we have 5 equations and four unknowns and it seems that we should be able to solve them. But Equation (1) is no more than (2) + (4) or (3) + (5). It gives no new information. There are solutions in fact; for $a = 1, 2, 3$ or 0 .



It is interesting to extend the investigation to other polygons. You will find that where the number (n) of sides is odd you will get a unique solution. There will be $n + 1$ distinct equations and n unknowns. Where n is even the number of distinct equations is only n and it is likely that the solution is not unique. Can you see what determines the number of solutions?

(Page 7) Piles - Answers and Notes

2.	<u>Harder Games</u>	a)	3	6	3	b)	1	5	6	c)	3	7	2
			2	6	4		2	4	6		4	6	2
			4	4	4		4	4	4		4	4	4

(Page 8)	3.	<u>Harder still</u>	a)	1,3,8 → 1,2,9 → 3,4,7 → 4,4,4
			b)	2,8,2 → 1,9,2 → 1,7,4 → 4,4,4
			c)	3,4,5 → 4,3,5 → 4,1,7 → 4,4,4
			d)	3,6,3 → 2,7,3 → 4,7,1 → 4,4,4
			e)	1,10,1 → 2,9,1 → 4,7,1 → 4,4,4
			f)	1,4,7 → 2,3,7 → 4,1,7 → 4,4,4

(Page 9)	<u>3 Piles Puzzle</u>	<u>Puzzle 1</u>	4 4 4	<u>Puzzle 2:</u>	4 moves
			4 3 5		4 4 4
			4 5 3		3 5 4
			1 8 3		5 3 4
			1 4 7		8 0 4
			1 9 2		12 0 0
			1 3 8		
			1 10 1		
			1 2 9	<u>Puzzle 3</u>	Not possible
			1 11 0		
			11 1 0		
			0 12 0		
			0 0 12		

(Page 10) Piles : Notes on Investigation

(1) These are two ways to find which arrangements can be made into equal piles in 3 moves:

a) Go through all possible arrangements and try and solve each one.

b) Reverse the problem. This is a more effective method. Here we start with 3 equal piles, the first move is to move 3, the second to move 2, the third to move 1 stone. The first move leads to 7,1,4. The second move to: 5,3,4 or 5,1,6 or 9,1,2 or 7,3,2.

The third move leads to:

4,4,4; 4,3,5; 6,2,4; 6,3,3; 6,0,6; 6,1,5
5,2,5; 4,1,7; 5,0,7; 8,1,3; 10,0,2; 9,0,3;
10,1,1; 9,2,1; 7,2,3; 8,2,2.

These 16 arrangements are therefore all the possible starting arrangements that can be reformed into 3 equal piles in 3 moves.

(2) There are 19 different ways that 12 stones can be arranged in 3 piles. There are 3, therefore, which are not reached when we reverse the problem. They are 12,0,0; 11,1,0 and 8,4,0. These are impossible starting arrangements.

It is obvious why 12,0,0 and 11,1,0 cannot be equalized for the maximum number of stones we can move is 6 and $12 - 6 > 4$, $11 - 6 > 4$. The case of 8,4,0 is more puzzling. If we allowed negative piles it would be possible!

8,4,0 $\rightarrow$ 9,4,-1 $\rightarrow$ 7,4,1 $\rightarrow$ 4,4,4.

(3) With 3 moves and 9 stones all starting arrangements can be made into 3 piles of 3 stones.

With 4 moves and 15 stones all starting arrangements can be made into 3 piles of 5 stones.

(4) General Rule If we allow N moves the total number, T , of stones that can be moved is $\frac{N(N+1)}{2}$.

S , The total number of stones in the game then must not be greater than $\frac{3T}{2}$.

$$\therefore S = \frac{3N(N+1)}{4}$$

Now $N(N+1)$ is not always divisible by 4. Only when N itself or $N+1$ is a multiple of 4.

$$\therefore N = 3 \text{ or } 4$$

$$N = 7 \text{ or } 8$$

$$N = 4a - 1 \text{ or } 4a \text{ will give solutions for } S.$$

In cases (1) and (2) above N took the values 3 and 4. The next case $N = 7$ gives $T = 28$ and $S = 42$ ie 42 stones and 7 moves.

(Page 11)

Answers 12 stones

8, 3, 1 $\rightarrow$ 7, 3, 2 $\rightarrow$ 4, 6, 2 $\rightarrow$ 4, 4, 4
or 8, 3, 1 $\rightarrow$ 5, 6, 1 $\rightarrow$ 4, 6, 2 $\rightarrow$ 4, 4, 4
or 8, 3, 1 $\rightarrow$ 8, 2, 2 $\rightarrow$ 6, 4, 2 $\rightarrow$ 4, 4, 4

a) 2, 9, 1 $\rightarrow$ 2, 8, 2 $\rightarrow$ 2, 6, 4 $\rightarrow$ 4, 4, 4
or 2, 9, 1 $\rightarrow$ 4, 7, 1 $\rightarrow$ 4, 6, 2 $\rightarrow$ 4, 4, 4

b) 2, 7, 3 $\rightarrow$ 4, 5, 3 $\rightarrow$ 4, 2, 6 $\rightarrow$ 4, 4, 4
or 2, 7, 3 $\rightarrow$ 4, 7, 1 $\rightarrow$ 4, 6, 2 $\rightarrow$ 4, 4, 4
or 2, 7, 3 $\rightarrow$ 2, 4, 6 $\rightarrow$ 4, 4, 4

Answers 24 stones

a) 3, 8, 13 $\rightarrow$ 6, 8, 10 $\rightarrow$ 12, 8, 4 $\rightarrow$ 8, 8, 8
b) 7, 8, 9 $\rightarrow$ 14, 8, 2 $\rightarrow$ 12, 8, 4 $\rightarrow$ 8, 8, 8
c) 2, 7, 15 $\rightarrow$ 2, 14, 8 $\rightarrow$ 4, 12, 8 $\rightarrow$ 8, 8, 8

Hard ones

a) 16, 5, 3 $\rightarrow$ 13, 5, 6 $\rightarrow$ 8, 10, 6 $\rightarrow$ 8, 1, 12 $\rightarrow$ 8, 8, 8
b) 13, 9, 2 $\rightarrow$ 4, 18, 2 $\rightarrow$ 8, 14, 2 $\rightarrow$ 8, 12, 4 $\rightarrow$ 8, 8, 8
c) 10, 9, 5 $\rightarrow$ 10, 4, 10 $\rightarrow$ 10, 8, 6 $\rightarrow$ 4, 8, 12 $\rightarrow$ 8, 8, 8
d) 14, 5, 5 $\rightarrow$ 9, 10, 5 $\rightarrow$ 4, 10, 10 $\rightarrow$ 8, 6, 10 $\rightarrow$ 8, 12, 4 8, 8, 8

(1) 12 stones: To find all possible starting arrangements, once it is a good idea to reverse the problem. Then we start with 3 equal heaps of 4 stones and each move becomes a splitting move, half a heap is moved to another heap.

The first move gives us 6,4,2.

The second move gives 5 new arrangements: 3,7,2;

3,4,5; 8,2,2; 7,4,1; 6,5,1

The third move gives 3 new arrangements:

3,8,1; 5,2,5; 9,1,2.

The fourth move gives only 1 new arrangement: 10,1,1.

Including 4,4,4 this gives 11 starting arrangements which can be equalized. Compare this with the total number of possible arrangements of 12 stones in 3 piles:

<u>12,0,0</u>	<u>11,1,0</u>	<u>10,2,0</u>	<u>10,1,1</u>	<u>9,3,0</u>	<u>9,2,1</u>
<u>8,4,0</u>	<u>8,3,1</u>	<u>8,2,2</u>	<u>7,5,0</u>	<u>7,4,1</u>	<u>7,3,2</u>
<u>6,6,0</u>	<u>6,5,1</u>	<u>6,4,2</u>	<u>6,3,3</u>	<u>5,5,2</u>	<u>5,4,3</u>

and 4,4,4.

Eight arrangements can not be made; seven of these because they contain a zero eg. 8,4,0 and 11,1,0 and a zero heap can never grow. The other impossible starting arrangement is 6,3,3.

(2) 24 stones: Once again all starting arrangements containing zero are impossible. If we follow the method used in (1) of reversing the problem we find that 12,6,6 and the results of splitting 12,6,6 ie 12,9,3; 15,6,3; 18,3,3 cannot be made.

∴ these are impossible starting arrangements.

In general if the total number of stones is $4a$ then a starting arrangement of $2a, a, a$ will be impossible to equalize.

(3) The number of stones in each heap at the end of the puzzle must be a member of the series: $1, 2, 4, 8, 16 \dots 2^N$.

There are 3 heaps so the total number of stones, T , is given by

$$T = 3 \times 2^N.$$

But T must not be a multiple of 4 or, as found in (2), there will be at least one impossible starting arrangement.

∴ $N = 0$ or $N = 1$

and $T = 3$ or 6 .

Odds and Evens

Answers and Notes

In most cases there are several possible answers.

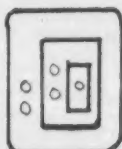
(Page 13) Puzzle 1



(Page 14) Puzzle 2



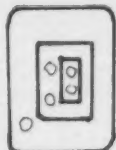
(Page 14) Puzzle 3



Puzzle 4



Puzzle 5



Puzzle 6



Always start by filling C first.

(Page 15) 7 stones A



B



C



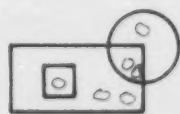
D



(Page 16) 6 stones A



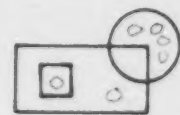
B



C

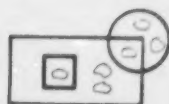


D

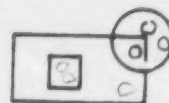


(Page 16)

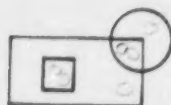
E



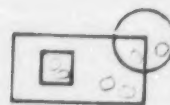
F



G



H



(Page 17) Crossing to the island

(1)

(1,2,4) →

(2)

(2,3,5) →

(3)

Use the Even rule.

← (1)

← (2)

(2,3,5) →

(1,3,5) →

(2,1,7) →

← (2)

← (1)

← (2)

(2,1,7) →

(1,6) →

(2,4,6) →

← (2)

(2,4,6) →

← (2)

(2,8) →

With 6 stones use the Odd rule.
With 7 stones use the Even rule.
With 8 stones use the Even rule.
With 9 stones use the Odd rule.

A pattern is starting to appear. To get more clues it is a good idea to look at the simpler cases. What happens with 5 stones, with 4 stones?
Now try and predict what will happen with 10 stones.
Next we should try and see why the pattern seems to be Odd, Odd, even, even, Odd, Odd. You probably noticed when doing the puzzles that you had to be most careful with the Odd numbered stones. It is the number of Odd stones in the village that tell us which rule to use.

With 5 stones (1) (2) (3) (4) (5) there are 3 odds. Now anyone of these odd ones can be boat man. The two other odds can be taken on 1 journey because $\text{odd} + \text{odd} + \text{odd} = \text{odd}$. Then the evens are no problem.

But with 7 stones (1) (2) (3) (4) (5) (6) (7) there are 4 odds (1) (3) (5) (7). One can be boatman, 2 can be taken as a pair but there is no way that the last odd number can be carried for the boatman is odd too, and $\text{odd} + \text{odd} = \text{even}$.

So if there is an odd number of odd stones in the village they can be carried using the odd rule.
With 5 or 6 stones we get an odd number of odds.
With 9 or 10 stones we do as well.
With 13 or 14 stones we do as well.

So for all these the odd rule must be used.
These numbers (5,6) (9,10) (13,14) are all one or 2 more than multiples of 4. They can be given the general name $4a + 1$ and $4a + 2$ (when $a = 1$ we get 5 and 6 when $a = 2$ we get 9 and 10).

The other numbers will need the even rule. (7,8) (11,12) (15,16). These are the multiples of 4 and one less than the multiples of 4. Their general name is $4a$ and $4a - 1$.

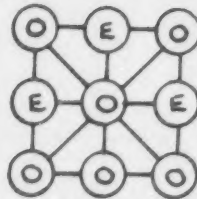
(1)



(2)



(3)



(4) Not possible

These solutions have connections with the magic square. If 9 consecutive numbers starting with an odd number are to be placed so that all lines sum to the same total then this sum will be odd, and the odd numbers must be placed in the positions shown in our solution to puzzle 3.

If 5 out of the nine numbers are even then the magic number (the sum of each line) must be even too and the even numbers must take the middle positions leaving the odds to fill the corners.

Is it possible to have a magic square in which 3 of the numbers are odd?

Notes on Investigation

- (1) If the aim is to get an ODD total.

<u>Player A</u>	<u>Player B</u>	<u>Stones</u>
E		OEEO
E	0	EOEO
EE	0	OEO

Player A is now certain to be able to pick an odd on this third go and finish with odd + even + even = odd.
This is the case even if player B takes an even on his first go.

if the aim is to get an EVEN total then first player takes an odd.

If second player takes an even, first player takes another odd. There are now two evens and one odd left and on his third take the first player is bound to be able to take another even to end with odd + odd + even = EVEN.

- (2) The game cannot be played with 8 stones, for both players will end with an even total or both with an odd total.
The sum of 8 consecutive numbers is even (four will be odd, four even and the sum even).
Breaking this into 2 sub totals can only create two even or two odd sub totals. Any number of stones which is a multiple of 4 will be unsuitable for this game.
- (3) Ten stones will work. The sum of 5 odd and 5 even numbers is an odd number.
An odd number can only be sub-divided into odd number and an even number. One player will always be the winner.
- (4) With 5 stones we get a very interesting result. The first player can always win if the aim is to get an even total but if the aim is to get an odd total the second player should win!

By the way, can we play the game with 5 stones numbered from 2 to 6?

Puzzle 1

1,2 →	
← 1	2
3 →	
← 2	3
1,2 →	

(Page 24)

Puzzle 2

1,3 →	
← 1	3
4 →	
← 3	4
1,3 →	
← 1	3,4
1,2 →	

(Page 24) Puzzle 3

1,2,4 →	
← 1	2,4
7 →	
← 2	4,7
1,2,3 →	
← 1	2,3,4,7
1,6 →	
← 1	2,3,4,6,7
1,5 →	

Puzzle 4

1,1,2 →	
← 1	1,2
4 →	
← 1	2,4
1,1,2 →	
← 1	1,2,2,4
1,3 →	

Puzzle 5

1,2,2 →	
← 1	2,2
5 →	
← 2	2,5
1,2,2 →	

Puzzle 6

1,4 →	
← 1	4
3,1 →	
← 4	1,3
2,4 →	

Puzzle 7

1,3,5 →	
← 1	3,5
2,4,6 →	
← 6	2,3,4,5
1,6 →	

(page 26)

Puzzle 8

2,4 →	
← 4	2
1,4 →	
← 1	2,4
1,3 →	

2,4 →
 ← 2, 4
 1,3 →
 ← 3 1,4
 3,5 →
 ← 4 1,3,5
 4,2 →

Notes on Investigation

(1) 7 units. This is 3 full loads (3×3) less 2×1 as 1 rows the boat back twice.

(2) If the boat can carry W units and makes N journeys, then the total weight that can be carried, T , is

$$T = \frac{W(N+1)}{2} - \frac{(N-1)}{2}$$

$\frac{N+1}{2}$ is the number of left to right crossings.

$\frac{N-1}{2}$ is the number of right to left crossings.

N must be odd as the boat ends on the right side of the river.

(Page 26) Sliding Puzzles - Answers: H : 13 moves

(Page 27) A : 14 moves K : 8 moves
 L : 22 moves T : 10 moves P : 10 moves
 (Page 28) Big H : 28 moves
 (Page 29) Big A : 30 moves; Big K : 20 moves; Big P : 14 moves

(page 34) 4 Stones in a Line

1. 17 moves 2. 12 moves 3. 10 moves
 4. 12 moves 5. 12 moves
 (Page 35) 6. 16 moves 7. 20 moves 8. 16 moves
 9. 25 moves 10. 12 moves 11. 20 moves

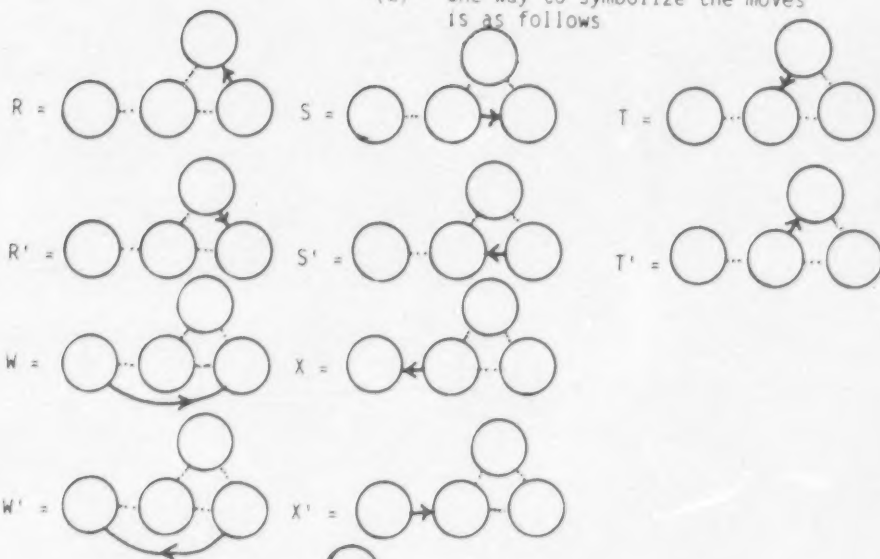
Sliding Puzzles

(Page 39) Notes on Investigation

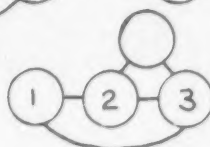
(1) Yes, it is. The 6 combinations are:

1,2,3	1,3,2
2,1,3	2,3,1
3,1,2	3,2,1

(2) One way to symbolize the moves is as follows

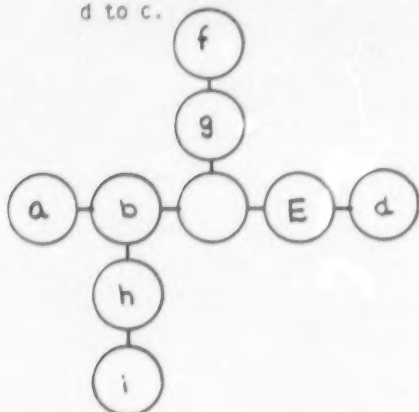


Starting with

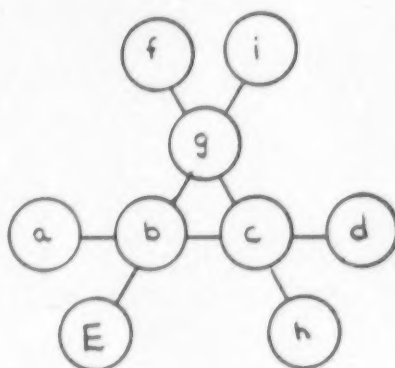


RST	→	132
RWXT	→	321
RST	→	312
T'X'W'R'	→	213
RST	→	231

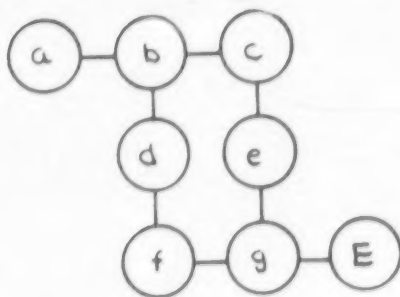
- (5) b to E, i to b, a to c
E to b, f to c, b to E
d to c.



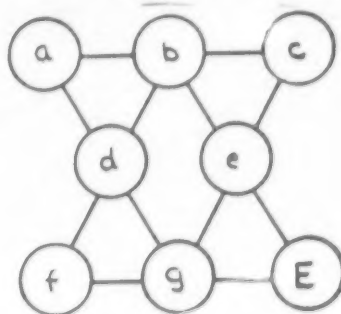
- (6) g to E, h to g, i to b,
a to c, d to b, E to g,
f to c.



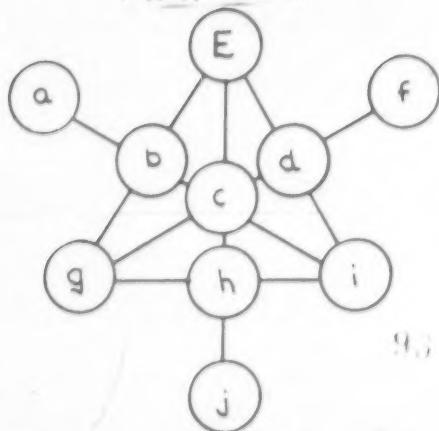
- (7) f to E, c to g, a to c
E to f, f to b, c to a



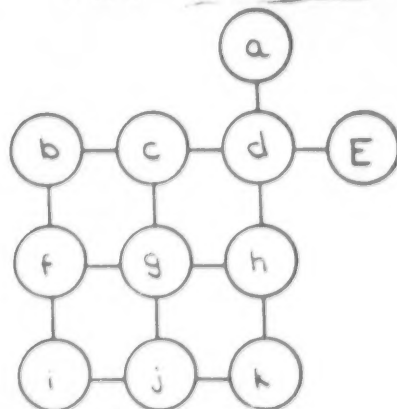
- (8) f to E, c to g, a to c
E to f, f to b, c to a



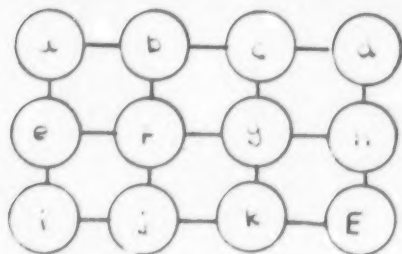
- (9) g to E, i to b, j to c
E to h, a to c, h to E
f to c, E to h.



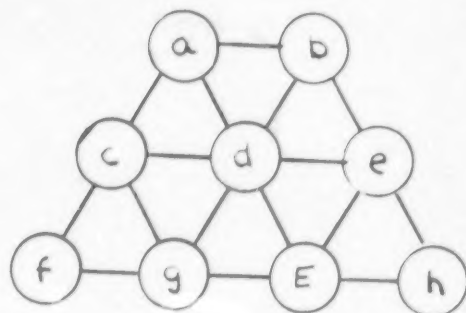
- (10) c to E, k to d, i to k,
f to h, E to c, k to d,
a to h, h to d, h to a.



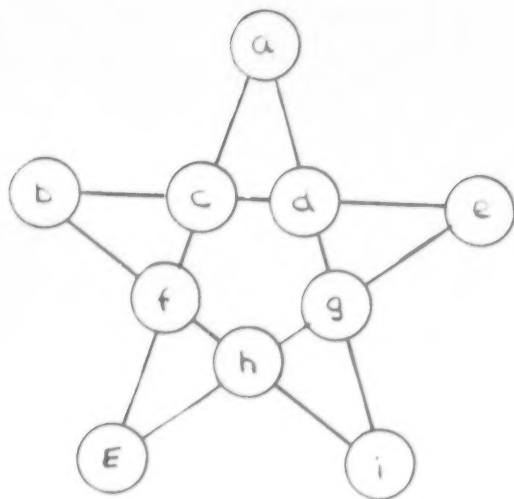
(Page 42) (11) d to E, b to d, j to b,
E to j, a to c, i to a,
d to b, a to c, c to k
j to E.



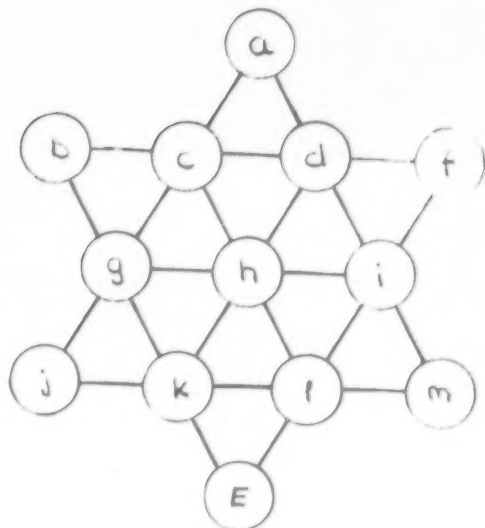
(12) f to E, a to f, h to g
f to E, b to g, g to h
h to b.



(13) c to E, i to f, d to i
b to h, E to g, i to d
e to c, a to f,
e to c, a to f.



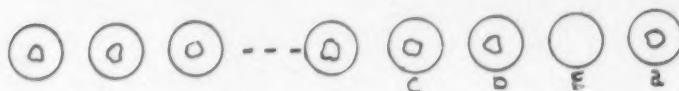
(14) i to E, j to e, c to j,
E to i, f to c, m to d,
a to g, b to k, j to f,
f to c, c to f.



- (1) 5 stone solitaire can only be solved if the empty circle is one away from the end.



- (2) If the number of circles is odd and the number of stones even, then the puzzle is impossible. If the number of circles is even and the number of stones is odd then the puzzle is possible. The empty circle should be one from the end.



With an odd number of stones there will be an even number on the left of E.

If C jumps b and then a jumps c we have:



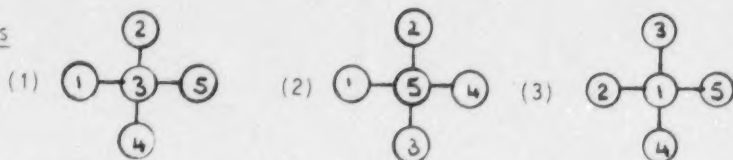
The number of stones on the left of E is still even. On the right of E there is still one. The total number has been reduced by 2. continuing this process is bound to lead to a repeated reduction by 2 stones and a similar position. In the end only one stone remains.

In the same way if the number of stones on the left of E is odd the situation will reduce to :



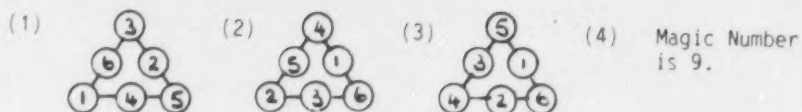
(Page 43) Magic Shapes - Answers and Notes

Magic Cross

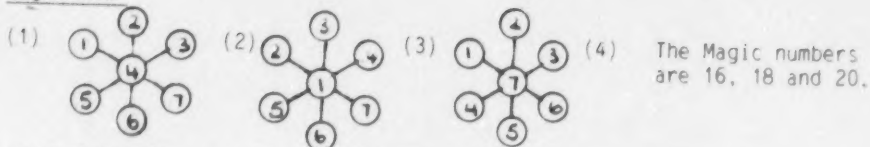


- (4) The magic numbers are 11, (2 in the centre)
12 (4 in the centre)
and 13 (6 in the centre).

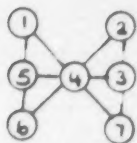
(Page 44) Magic Triangle



(Page 45) Magic Star



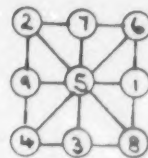
(Page 46) Magic Butterfly



Magic Diamond



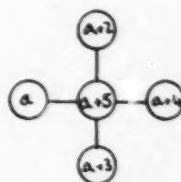
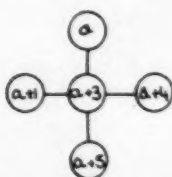
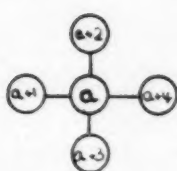
(Page 47) Magic Square



- (1) In the Magic Cross the 3 numbers than can be placed in the centre are:

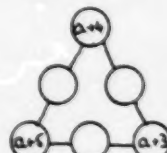
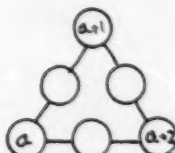
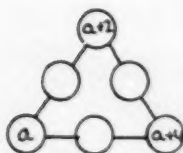
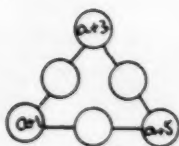
The first number in the sequence.
The middle number in the sequence.
The last number in the sequence.

If the numbers are $a, a + 1, a + 2, a + 3, a + 4$
then the magic numbers will be $3a + 5,$
 $3a + 4$ and $3a + 5, 3a + 6, 3a + 7$



- (2) In the Magic triangle to make it magic
a) place all the odd numbers in the corners or
b) place all the even numbers in the corners or
c) place the 3 lowest numbers in the corners or
d) place the 3 highest numbers in the corners.
If the numbers are $a, a + 1, a + 2, a + 3, a + 4,$
 $a + 5$ then the Magic numbers will be:

$3a + 10, 3a + 11, 3a + 9$ and $3a + 12.$



Number Puzzles - Answers

$$1 + 2 + 3 = 6$$

$$1 + 3 + 4 = 8$$

$$1 + 2 + 4 = 7$$

$$2 + 3 + 4 = 9$$

$$2 + 3 - 1 = 4$$

$$2 + 3 - 4 = 1$$

$$2 + 4 - 1 = 5 \quad \text{or} \quad 3 + 4 - 2 = 6$$

$$2 + 4 - 3 = 3 \quad \text{or} \quad 1 + 4 - 2 = 3$$

$$1 \times 4 + 2 = 6$$

$$2 \times 3 + 1 = 7$$

$$3 \times 4 + 1 = 13$$

$$2 \times 4 + 3 = 8$$

$$2 \times 3 - 1 = 5$$

$$3 \times 4 - 2 = 10$$

$$2 \times 4 - 1 = 7$$

$$2 \times 3 - 4 = 2$$

Notes on Investigation (1) 4 possible answers; 18,19,20,21.

(2) 8 possible answers. Some answers can be made in more than 1 way eg.

$$5 + 6 - 7 = 4 \quad \text{and} \quad 5 + 7 - 8 = 4.$$

The possible answers with the number of ways each can be made are:

3	1 way
4	2 ways
5	1 way
6	2 ways
7	2 ways
8	1 way
9	2 ways
10	1 way

Notice the symmetry of the pattern.

(3) using 4 consecutive numbers the smallest possible answer will be given by:

$$(a) + (a + 1) - (a + 3) = a - 2$$

The largest possible answer will be given by

$$(a + 2) + (a + 3) - (a) = a + 5$$

(Page 51) Adding Puzzles (A) - Answers

$\begin{array}{r} 12 \\ +34 \\ \hline 46 \end{array}$	a) $\begin{array}{r} 24 \\ +13 \\ \hline 37 \end{array}$	b) $\begin{array}{r} 41 \\ +23 \\ \hline 64 \end{array}$	c) $\begin{array}{r} 42 \\ +13 \\ \hline 55 \end{array}$
---	--	--	--

Subtraction Puzzle (A) - Answers

a) $\begin{array}{r} 42 \\ -31 \\ \hline 11 \end{array}$	b) $\begin{array}{r} 43 \\ -21 \\ \hline 22 \end{array}$	c) $\begin{array}{r} 43 \\ -12 \\ \hline 31 \end{array}$	d) $\begin{array}{r} 34 \\ -21 \\ \hline 13 \end{array}$	e) $\begin{array}{r} 42 \\ -13 \\ \hline 29 \end{array}$	f) $\begin{array}{r} 41 \\ -32 \\ \hline 9 \end{array}$
--	--	--	--	--	---

(Page 52) Adding Puzzles (B)

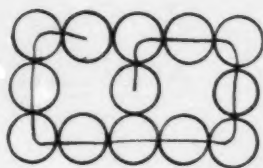
$\begin{array}{r} 123 \\ +456 \\ \hline 579 \end{array}$	a) $\begin{array}{r} 246 \\ +531 \\ \hline 777 \end{array}$	b) $\begin{array}{r} 265 \\ +134 \\ \hline 399 \end{array}$	c) $\begin{array}{r} 125 \\ +364 \\ \hline 489 \end{array}$	d) $\begin{array}{r} 562 \\ +431 \\ \hline 993 \end{array}$
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Subtraction Puzzle (B)

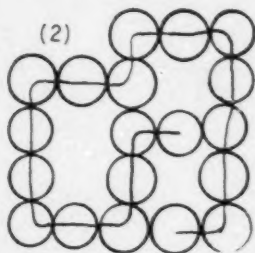
a) $\begin{array}{r} 456 \\ -123 \\ \hline 333 \end{array}$	b) $\begin{array}{r} 654 \\ -123 \\ \hline 531 \end{array}$	c) $\begin{array}{r} 264 \\ -153 \\ \hline 111 \end{array}$	d) $\begin{array}{r} 253 \\ -146 \\ \hline 107 \end{array}$	e) $\begin{array}{r} 613 \\ -425 \\ \hline 188 \end{array}$	f) $\begin{array}{r} 621 \\ -345 \\ \hline 276 \end{array}$
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(Page 54) Path Puzzles. There are several possible answers.

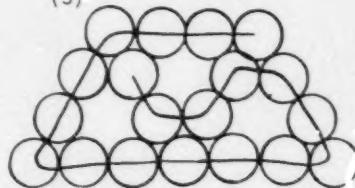
(1)



(2)



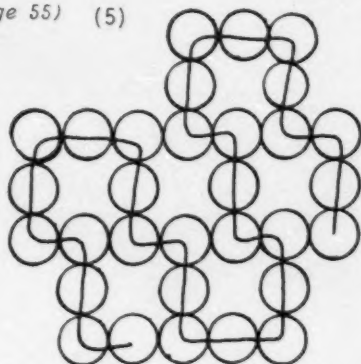
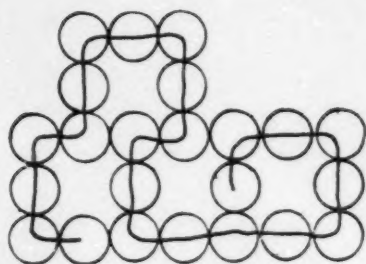
(3)



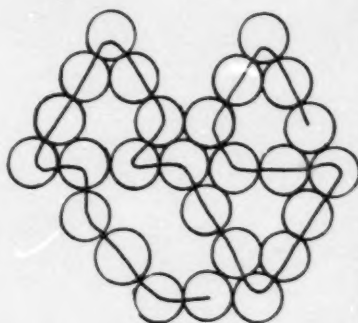
(4)

(Page 55)

(5)

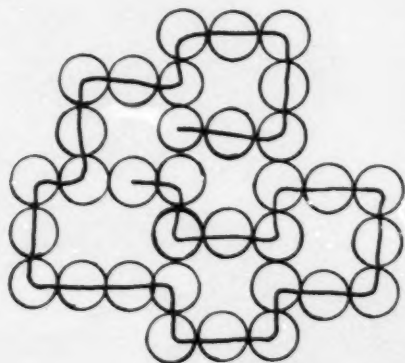


(6)

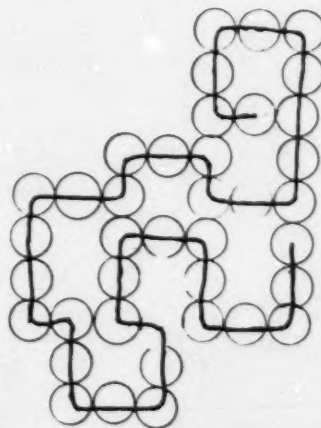


(Page 55)

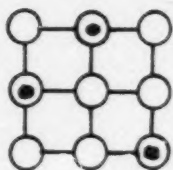
(7)



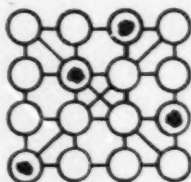
(8)



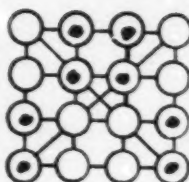
(1)



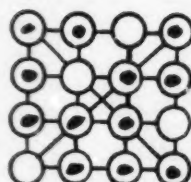
(2)



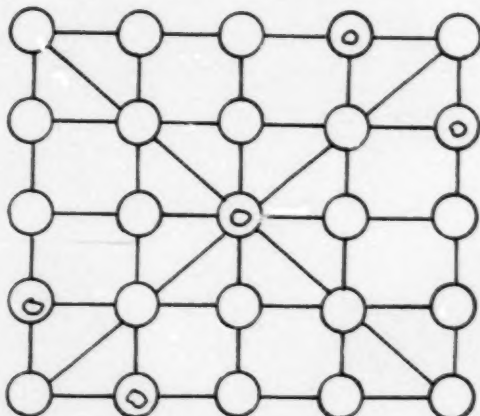
(3)



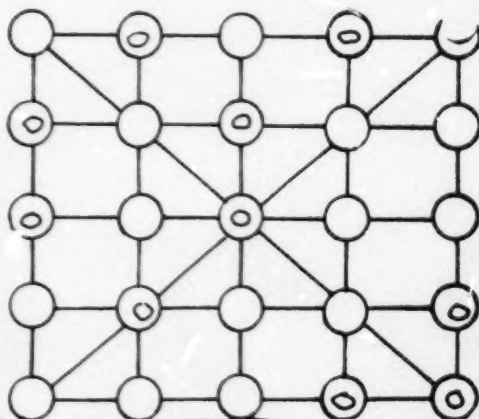
(4)



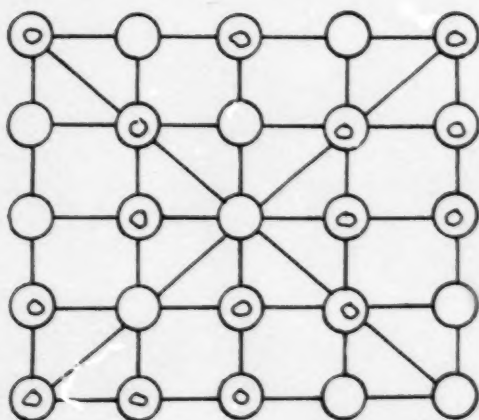
(5)



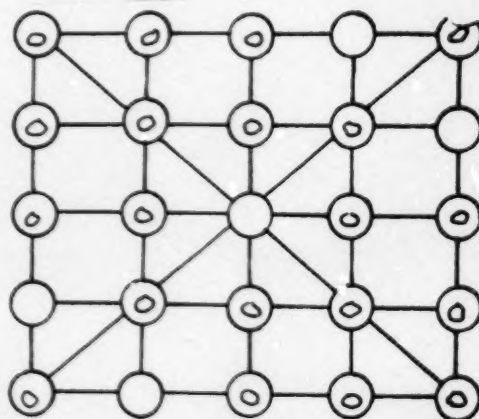
(6)



(7)



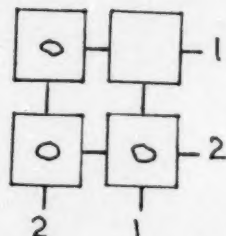
(8)



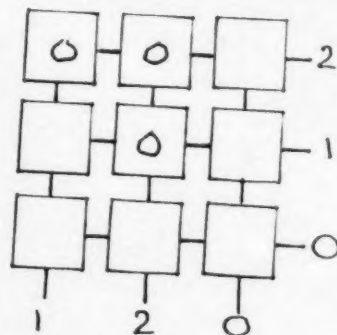
You will notice that Puzzles 2 and 4 are really both the same. One asks for 1 stone per line, the other for 1 space per line. In the same way 5 and 8, 6 and 7 are the same.

2 to 1 to 3 to 2 to 2 to FINISH.

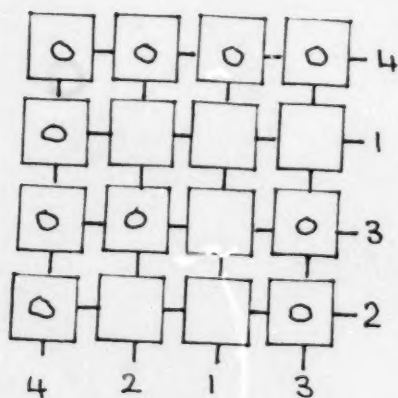
(a)



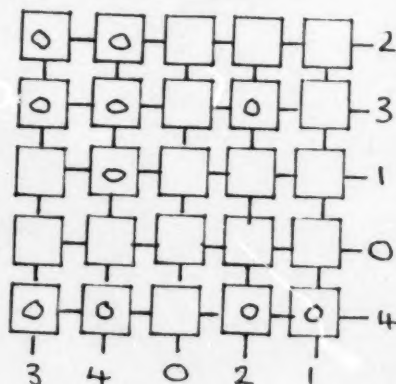
(b)



(c)



(d)



Exchange Puzzles

Notes on Investigation

- (1) 6 moves
- (2) 18 moves
- (3) 34 moves

If N is the number of stones on one side, then the number of moves, M , is given by:

$$M = 2N^2 + 6N - 2.$$

Each stone has to move $N + 1$ steps across.
There are $2N$ stones giving $2N(N + 1) = 2N^2 + 2N$ moves.
 $N - 1$ of the stones have to move in and out of the top circles 4 times giving $4(N - 1)$ moves.
One stone moves in and out of a top circle just once giving 2 moves.

$$\text{Total} = 2N^2 + 2N + 4N - 4 + 2 = 2N^2 + 6N - 2$$